



厦门大学马来西亚分校
陈景润杯中学数学比赛



2017 年第 1 届陈景润杯中学数学比赛

~ 初阶组 ~

日期: 2017 年 4 月 1 日

Date: 1st April 2017

时间: 下午 2 时至 4 时

Time: 2:00 p.m. to 4:00 p.m.

考生须知

Instructions and Information

1. 本试卷共有 30 题。

This paper contains 30 questions.

- 第 1 至第 10 题, 选择题, 每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

- 第 11 至第 30 题, 问答题, 每题请填入一个 -1000 至 1000 之间的整数为答案。

Question 11 to Question 30, short questions. For each question, please fill in an integer between -1000 and 1000 as answer.

- 第 11 至第 20 题每题 5 分。

Question 11 to Question 20, each question carries 5 marks.

- 第 21 至第 25 题每题 6 分。

Question 21 to Question 25, each question carries 6 marks.

- 第 26 至第 30 题每题 8 分。

Question 26 to Question 30, each question carries 8 marks.

2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题, 只需填写 A, B, C, D 或 E 作为答案。每题只能填入一个答案, 否则以答错论。

Please use 2B lead pencils to put your answer to each question in the appropriate space provided on the answer sheet. For a multiple choice question, you only need to put in A, B, C, D or E as answer. You can only give an answer to each question, otherwise it is considered that you answer that question incorrectly.

3. 所有的图形并没有按照比例作图, 只作为辅助之用。

All the diagrams are not drawn to scales. They are intended only as aids.

4. 不许使用计算器, 数学工具, 手机或其他计算器。

No calculators, maths stencils, mobile phones or other calculating aids are permitted.

5. 在答案纸上清楚写上姓名, 考生编号, 学校名称及在学年级。

Write your name, IC number, school name and year of study clearly on the answer sheet.

6. 在监考老师宣布比赛开始之后, 才可以翻开此考卷开始作答。

You can only open this question booklet to start answering questions after the invigilator announce the beginning of the competition.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中, $\lfloor x \rfloor$ 表示小于或等于 x 的最大整数。

例如: $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$ 。

In this paper, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题, 选择题, 每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 一个立方体的六个面分别写有数字 1, 2, 3, 4, 5, 6。有多少种方法可以将它的其中两个不相邻的面漆成红色?

The six faces of a cube are numbered 1, 2, 3, 4, 5, 6 respectively. How many ways are there to paint two faces of the cube red so that no two red faces are adjacent?

A. 0 B. 1 C. 2 D. 3 E. 6

2. 有多少个自然数是质数, 但它的个位数不是 1, 3, 7 或 9?

How many natural numbers are prime numbers whose last digit is not 1, 3, 7, or 9?

A. 无穷多个 Infinitely many

B. 0

C. 1

D. 2

E. 3

3. 如果 5 个人在 5 天内可制作 5 件玩具, 需多少个人才可在 50 天内制作 50 件玩具?

If 5 people can make 5 toys in 5 days, how many people would it take to make 50 toys in 50 days?

A. 1 B. 5 C. 25 D. 50 E. 500

4. 一等边三角形的面积为 10 m^2 。若它每边的长都增加至原来的三倍，则其面积变为多少？

The area of an equilateral triangle is 10 m^2 . What is the area of the triangle after the length of each side is tripled?

- A. 10 m^2 B. 30 m^2 C. 90 m^2 D. 270 m^2 E. 300 m^2

5. 有一个立方体及一粒球。若立方体的边长与球的直径一样，则

There is a cube and a ball. If the side length of the cube is the same as the diameter of the ball, then

- A. 球的体积比立方体的体积大。

The volume of the ball is larger than the volume of the cube.

- B. 立方体的体积比球的体积大。

The volume of the cube is larger than the volume of the ball.

- C. 立方体与球的体积一样。

The volume of the cube is the same as the volume of the ball.

- D. 所给的资讯不足以判断球与立方体何者有较大的体积。

The given information is not enough to determine whether the cube or the ball has larger volume.

- E. 立方体的边长与球的直径不可能一样。

It is impossible that the side length of the cube is the same as the diameter of the ball.

6. 由 2, 5, 6 这三个数字所组成的数字不重复的三位数中，其中有几个可以被 3 整除？

Among all the three-digit numbers formed by 2, 5, 6 that have no repeated digits, how many of them is divisible by 3?

- A. 0 B. 1 C. 2 D. 3 E. 6

7. 若一长方形的长与宽都是不大于 7 的整数，则它的面积有多少个不同的可能值？(正方形也是长方形。)

If the length and width of a rectangle are both integers not exceeding 7, how many different numbers could be the area of this rectangle? (A square is also a rectangle.)

A. 23 B. 25 C. 28 D. 42 E. 49

8. 若 $2x$ 及 $\frac{55}{x}$ 都是正整数，则 x 有几个可能值？

If $2x$ and $\frac{55}{x}$ are both positive integers, how many possible values can x assume?

A. 1 B. 4 C. 6 D. 7 E. 8

9. 已知一三角形三边的长都是整数。若其中两边的长分别为 2011 及 2017，则第三边的长的可能值有几个？

Given that all three sides of a triangle have integer lengths. If two of the sides have length 2011 and 2017 respectively, find the possible number of values for the length of the third side.

A. 2014 B. 2016 C. 4020 D. 4021 E. 4027

10. 由 1 000 000 到 9 999 999 的整数中，有多少个含有 0, 1, 2, 3, 4, 5, 6 这 7 个数字的每一个各一次？

Among the integers from 1 000 000 to 9 999 999, how many of them has each of the digits 0, 1, 2, 3, 4, 5, 6 exactly once?

A. 7000 B. 5040 C. 4320 D. 3500 E. 2800

第 11 至第 20 题，问答题，每题 5 分。

Question 11 to Question 20, short questions, each question carries 5 marks.

11. 2017^{2017} 的个位数是什么？

What is the last digit of 2017^{2017} ?

12. 两支蜡烛，一支红色，一支黄色。它们一样长，且同一时间被点燃。红色蜡烛烧了 6 小时才烧完，黄色蜡烛烧了 3 小时就烧完。若点燃 x 小时后，红色蜡烛的长度是黄色蜡烛的 2 倍，求 $60x$ 的值。

There is one red candle and one yellow candle. The two candles have the same length and they are lit at the same time. The red candle takes 6 hours to burn completely, and the yellow candle takes 3 hours to burn completely. If the length of the red candle is twice that of the yellow candle x hours after they are both lit, find the value of $60x$.

13. 一个课程一共有 n 次考试，已经考了 $(n-1)$ 次。若小明在最后一次考试考 100 分，则他在 n 次考试的平均分数是 90 分。若他在最后一次考试考 60 分，则他在 n 次考试的平均分数是 85 分。求 n 的值。

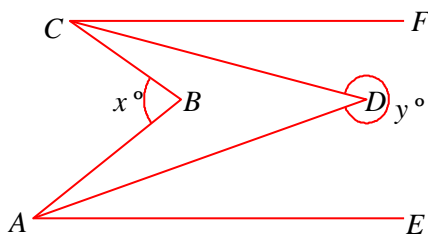
A course has n exams, $(n-1)$ of them have been held. If Xiao Ming scores 100 in the last exam, his overall average in the n exams is 90. If he scores 60, his overall average is 85. Find the value of n .

14. 一间珠宝店所卖的每粒宝石的价格都是整数，它们的价格在每周一开始就调整。在这个星期，一粒红宝石与一粒蓝宝石的价格一样。红宝石这星期的价格比上星期的上涨了 10%，蓝宝石这星期的价格比上星期的下跌了 10%。求蓝宝石上星期的价格的最小可能值。

The prices of stones in a jewelry store are always positive integers. The prices are adjusted at the beginning of every week. This week, a red stone has the same price as a blue stone. For a red stone, its price has increased by exactly 10%. For a blue stone, its price has decreased by exactly 10%. What was the minimum possible price of a blue stone last week?

15. 下图中， $AE \parallel CF$ ， AD 平分 $\angle BAE$ ， CD 平分 $\angle BCF$ 。若 $y = 324$ ，求 x 。

In the figure below, $AE \parallel CF$, AD bisects $\angle BAE$, CD bisects $\angle BCF$. If $y = 324$, find x .

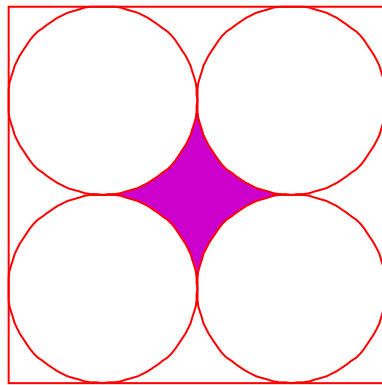


16. 小明每答对一题数学，他父亲就给他 RM5。小明每答错一题，他就需给他父亲 RM8。一次数学作业中共有100道题，扣掉了他父亲给他的，小明还需给他父亲 RM7。小明一共答对几题？

Xiao Ming's father will pay him RM5 for each correct answer he obtains in math homework. For each incorrect answer, Xiao Ming must pay his father RM8. There are 100 questions in a math homework. After subtracting the amount his father should pay him, Xiao Ming still need to pay his father RM7. How many correct answers did Xiao Ming get?

17. 下图所示是一个边长为 12 的正方形及四个全等的圆。若阴影部分的面积为 $a+b\pi$ ，其中 a, b 都是有理数。求 $a+b$ 的值。

The following figure shows a square with side length 12 and four congruent circles. If the area of the shaded region is $a+b\pi$, where a and b are rational numbers. Find the value of $a+b$.



18. 介于 1 与 2017 的整数中，有多少个可以被 7 整除，但不能被 5 整除？

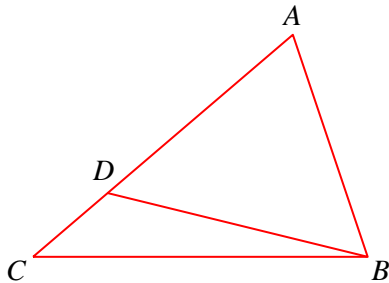
Among the integers from 1 to 2017, how many of them are divisible by 7, but not divisible by 5?

19. $5^{20} \times 4^{17}$ 这个数有几位数？

How many digits are there in the number $5^{20} \times 4^{17}$?

20. 下图中, D 是 AC 上的一点使得 $AD = AB$ 。若 $\angle ABC - \angle ACB = 42^\circ$, $\angle CBD = x^\circ$, 求 x 。

In the figure shown below, D is a point on AC such that $AD = AB$. If $\angle ABC - \angle ACB = 42^\circ$ and $\angle CBD = x^\circ$, find x .

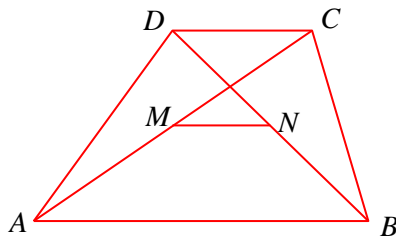


第 21 至第 25 题, 问答题, 每题 6 分。

Question 21 to Question 25, short questions, each question carries 6 marks.

21. 下图中, $ABCD$ 是一梯形, $AB > CD$, $AB \parallel CD$ 。 M 与 N 分别是线段 AC 与 BD 的中点。若 $AB = 1024$, $MN = 124$, 求 CD 的长。

In the figure shown below, $AB > CD$, $AB \parallel CD$. M and N are respectively the midpoints of the line segments AC and BD . If $AB = 1024$ and $MN = 124$, find the length of CD .



22. 衣橱里有 20 只蓝袜子与 10 只红袜子。一天晚上家里没电, 所以你看不到袜子的颜色。你至少需拿出几只袜子才能保证其中两只是蓝色, 两只是红色?

A closet contains 20 blue socks and 10 red socks. It is night, and the lights in your house are not working so you can't see the colours of the socks. How many socks must you remove from the closet to guarantee you have at least 2 blue socks and 2 red socks?

23. 已知一数列 a_1, a_2, a_3, \dots 满足以下的条件:

$$\text{对于所有的 } n \geq 1, a_1 + a_2 + a_3 + \dots + a_n = n^3.$$

求 a_{100} 的最后三位数。

Given that a sequence a_1, a_2, a_3, \dots satisfies the following conditions:

$$\text{For all } n \geq 1, a_1 + a_2 + a_3 + \dots + a_n = n^3.$$

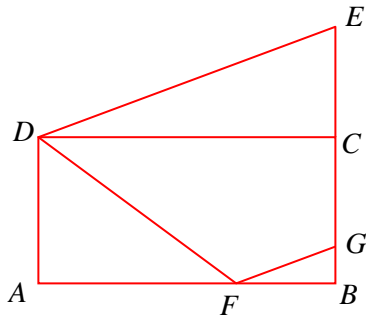
Find the last three digits of a_{100} .

24. 有多少个正整数 n 使得方程式 $x^2 - 1001x + n = 0$ 的根都是整数?

For how many positive integers n is such that the equation $x^2 - 1001x + n = 0$ has integer roots?

25. 下图中, $ABCD$ 是面积为 240 的长方形, ECG 是直线, $EC = CG$ 。 $DEGF$ 是梯形, $DE \parallel FG$ 。若 $\triangle ADF$ 为 90, 求 $\triangle BGF$ 的面积。

In the figure shown below, $ABCD$ is a rectangle with area 240, ECG is a straight line, $EC = CG$. $DEGF$ is a trapezium, $DE \parallel FG$. If the area of $\triangle ADF$ is 90, find the area of $\triangle BGF$.



第 26 至第 30 题, 问答题, 每题 8 分。

Question 26 to Question 30, short questions, each question carries 8 marks.

26. 设 n 是整数使得 $n+100$ 与 $n-24$ 都是平方数, 求 n 的最小可能值。

Let n be an integer such that $n+100$ and $n-24$ are perfect squares. Find the smallest possible value of n .

27. 求满足方程式 $\frac{|6n-10-n^2|+10n-6-n^2}{n^2-1} = \frac{1}{100}$ 的最小整数 n 。

Find the smallest integer n that satisfies the equation $\frac{|6n-10-n^2|+10n-6-n^2}{n^2-1} = \frac{1}{100}$.

28. 小明身上没钱，便拿了一张有四位数金额的支票到银行兑现。糊涂的出纳员将金额的四位数 $abcd$ 看成 $cdab$ ，而给了小明这金额的钱。小明没数就将钱拿走，到超市拿出其中的 RM 50 买了 RM 50 的东西后，才发现剩下的钱刚好等于支票上数额的 3 倍。问小明支票上原来的金额的最后三位数是什么？

Xiao Ming does not have money, so he took a check with amount a four-digit figure to the bank to cash. The confused bank teller did not read carefully, and read the amount on the check $abcd$ as $cdab$. Xiao Ming did not count the money and just took it away. After taking out RM 50 to buy something worth RM 50, Xiao Ming noticed that his remaining money is exactly three times as much as the original check. What was the last three digit of the original amount on the check?

29. 有 100 个士兵，在一场战役中，他们的受伤情况如下：70 人的眼睛受伤，75 人的耳朵受伤，85 人的脚受伤，80 人的手受伤。至少有几个人的眼睛，耳朵，脚及手四个部位都受伤？

A group of 100 soldiers suffered the following injuries in a battle: 70 soldiers suffered eye injuries, 75 soldiers suffered ear injuries, 85 soldiers suffered leg injuries, and 80 soldiers suffered arm injuries. What is the minimum number of soldiers who suffered injuries on all four parts: eyes, ears, legs and arms?

30. 将 n^3 个边长等于 1 的立方体拼合在一起形成一个边长为 n 的大立方体，然后将这个大立方体的其中 m 个面漆成红色。当红漆干了后，将大立方体又再分拆回原来的边长等于 1 的小立方体，结果发现有 210 个小立方体的任何面都没有红漆。有几个小立方体刚好有两个面有红漆？

Take n^3 unit cubes and assemble them to form a larger cube with side length n . Then m of the faces of the larger cube are painted red. After the paint dries, the larger cube is disassembled into the unit cubes and it is found that 210 of these unit cubes have no red paint in any of their faces. How many small cubes are there which has exactly two of its faces painted red?