厦门大学马来西亚分校

陈景润杯中学数学比赛

2018 年第 2 届陈景润杯中学数学比赛

~ 初阶组 ~

日期：2018 年 4 月 21 日

Date: 21st April 2018

时间：上午 10 时至中午 12 时

Time: 10:00 a.m. to 12:00 p.m.

考生须知

Instructions and Information

1. 本试卷共有 30 题。
   This paper contains 30 questions.
   - 第 1 题至第 10 题，选择题，每题 4 分。
     Question 1 to Question 10, multiple choice questions, each question carries 4 marks.
   - 第 11 题至第 30 题，问答题，每题的答案是一个介于 0 至 1000 之间的整数。
     Question 11 to Question 30, short questions. For each question, the answer is an integer between 0 and 1000.
     - 第 11 题至第 20 题每题 5 分。
       Question 11 to Question 20, each question carries 5 marks.
     - 第 21 题至第 25 题每题 6 分。
       Question 21 to Question 25, each question carries 6 marks.
     - 第 26 题至第 30 题每题 8 分。
       Question 26 to Question 30, each question carries 8 marks.
2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题，只需填写 A，B，C，D 或 E 作为答案。每题只能填入一个答案，否则以答错论。
   Please use 2B lead pencils to write your answer to each question in the appropriate space provided on the answer sheet. For a multiple choice question, you only need to write A, B, C, D or E as answer. Only one answer is accepted for each question, otherwise no credits would be given.
3. 所有的图形并没有按照比例作图，只作为辅助之用。
   All the diagrams are not drawn to scales. They are intended only as aids.
4. 不许使用计算器，数学工具，手机或其他计算器。
   No calculators, maths stencils, mobile phones or other calculating aids are permitted.
5. 在答案纸上清楚写上姓名，考生编号，学校名称及在学年级。
   Write your name, candidate number, school name and year of study clearly on the answer sheet.
6. 在监考老师宣布比赛开始之后，才可以翻开此考卷开始作答。
   You can only open this question booklet to start answering questions after the invigilator announces the beginning of the competition.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中，$\lfloor x \rfloor$ 表示小于或等于 x 的最大整数。

例如：$\lfloor 2 \rfloor = 2$，$\lfloor -2 \rfloor = -2$，$\lfloor 2.6 \rfloor = 2$，$\lfloor -2.6 \rfloor = -3$。

In this paper, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题，选择题，每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 求 0.0125×8000。

   Find 0.0125×8000.

   A. 0.1　　B. 1　　C. 10　　D. 100　　E. 1000

2. 化简 $3 + \dfrac{1}{7 + \dfrac{1}{16}}$。

   Simplify $3 + \dfrac{1}{7 + \dfrac{1}{16}}$.

   A. $\dfrac{361}{115}$　　B. $\dfrac{345}{113}$　　C. $\dfrac{363}{115}$　　D. $\dfrac{355}{115}$　　E. $\dfrac{355}{113}$

3. 今天是星期六，150 天之后是_____。

   Today is Saturday. 150 days later is _____.

   A. 星期天 Sunday　　B. 星期一 Monday　　C. 星期二 Tuesday　　D. 星期三 Wednesday　　E. 星期四 Thursday

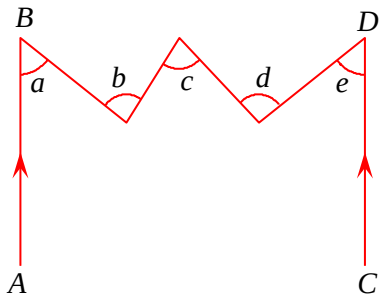
4. 一物品若按标价的 80%出售将获利 20%，若按标价出售，则获利的百分比是多少？
An item, if sold at 80% of the marked price, gives a profit of 20%. If the item is sold at the marked price, what is the profit in percentage?

A. 30% B. 40% C. 45% D. 50% E. 60%

5. 求 $\left\lfloor -\frac{100}{33} \right\rfloor + \left\lfloor \frac{100}{33} \right\rfloor$。
Find $\left\lfloor -\frac{100}{33} \right\rfloor + \left\lfloor \frac{100}{33} \right\rfloor$.

A. -2 B. -1 C. 0 D. 1 E. 2

6. 下图中，$AB // CD$。若 $a+b+c+d+e=310°$，$b+c+d=228°$，求 $\angle c$。
In the figure below, $AB // CD$. If $a+b+c+d+e=310°$, $b+c+d=228°$, find $\angle c$.



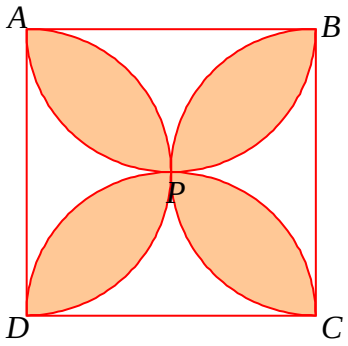
A. 114° B. 82° C. 76° D. 74° E. 73°

7. 已知两个正整数的乘积是 4320，最大公因数是 12。求这两个数的最小公倍数。
Given that the product of two positive integers is 4320, and their greatest common divisor is 12. Find the least common multiple of these two numbers.

A. 360 B. 720 C. 1080 D. 2160 E. 4320

8. 下图中，$ABCD$ 是边长为 1 的正方形，弧 $\overset{\frown}{APB}$，$\overset{\frown}{BPC}$，$\overset{\frown}{CPD}$ 及 $\overset{\frown}{DPA}$ 是半圆。若半圆 $\overset{\frown}{APB}$ 与直线 AB 所围成的面积是 S，求阴影部分的面积。

In the figure below, $ABCD$ is a square with side length 1. The arcs $\overset{\frown}{APB}$, $\overset{\frown}{BPC}$, $\overset{\frown}{CPD}$ and $\overset{\frown}{DPA}$ are semi-circles. If the area enclosed between the semi-circle $\overset{\frown}{APB}$ and the line AB is S, find the area of the shaded region.



A. $\frac{1}{2}$ B. $4S-1$ C. $1-2S$ D. S E. $\frac{2}{3}$

9. 不大于 200 的正整数中，有多少个可以被 2 或 5 整除?

Among the positive integers not larger than 200, how many of them are divisible by 2 or 5?

A. 100 B. 108 C. 120 D. 128 E. 140

10. 若 $x^3=2015\times2017\times2019+4\times2017$，求 x。

If $x^3=2015\times2017\times2019+4\times2017$, find x.

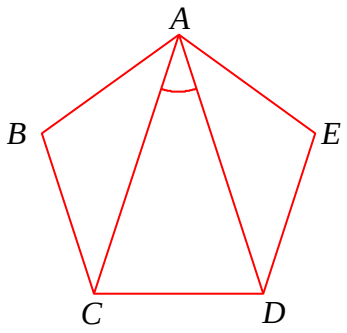
A. 2017 B. 2018 C. 2019 D. 2020 E. 2021

第 11 至第 20 题，问答题，每题 5 分。

Question 11 to Question 20, short questions, each question carries 5 marks.

11. 下图中， $ABCDE$ 是正五边形。若 $\angle CAD = x°$，求 x。

In the figure below, $ABCDE$ is a regular pentagon. If $\angle CAD = x°$, find x.



12. 求满足不等式 $4 \le 999 - 3x < 1000$ 的最大整数 x。

Find the largest integer x that satisfies the inequality $4 \le 999 - 3x < 1000$.

13. 求 $\dfrac{3\sqrt{3}+5}{3\sqrt{3}-5} + \dfrac{3\sqrt{3}-5}{3\sqrt{3}+5}$ 的值。

Find the value of $\dfrac{3\sqrt{3}+5}{3\sqrt{3}-5} + \dfrac{3\sqrt{3}-5}{3\sqrt{3}+5}$.

14. 若三位数 $\overline{2a7}$ 可以被 11 整除，求 a 的值。

If the three digit number $\overline{2a7}$ is divisible by 11, find the value of a.

15. 下课时，1001 位学生去食堂，每位男生吃了两碗饭，每位女生吃了一碗饭，结果这些学生一共吃了 1654 碗饭。问女生有几人?

During recess time, 1001 students went to canteen. Each boy ate two bowls of rice, and each girl ate one bowl of rice. A total of 1654 bowls of rice were consumed by the students. How many girls were there?

16. 有一群学生，其中 $\frac{1}{3}$ 是男生。女生中，有 $\frac{3}{8}$ 戴眼镜。若没有戴眼镜的女生有 45 人，问这群学生有多少人?

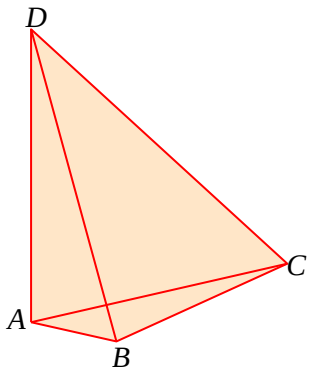
In a group of students, $\frac{1}{3}$ of them are boys, and $\frac{3}{8}$ of the girls wear glasses. If there are 45 girls who do not wear glasses, how many students are there in this group?

17. 架子上有 23 盒蓝色原子笔及 17 盒红色原子笔。每个盒子都密封着，盒子的表面完全一样，没有注明里面所含原子笔的颜色。林老师赶着去上课，却需要一盒红色原子笔。因此她打算先拿走若干盒原子笔，去到班上才打开找一盒红色的。林老师必须取走最少多少盒原子笔，才能保证至少有一盒是红色的?

There are 23 boxes of blue pens and 17 boxes of red pens on a shelf. Each box is sealed and the exterior of the boxes are identical. There is no indication on the boxes about the colour of the pens inside. Ms Lin is rushing to her class, but she needs a box of red pens. Therefore, she plans to take away a few boxes of pens, and only opens them in the class to find a box of red pens. At least how many boxes of pens should Ms Lin take with her in order to make sure that she has at least one box of red pens?

18. 如下图所示，$ABCD$ 是一四边形，其对角线 AC 与 BD 互相垂直。若 $AC=19$，$BD=22$，求四边形 $ABCD$ 的面积。

As shown in the figure below, $ABCD$ is a quadrilateral. The diagonals AC and BD are perpendicular. If $AC=19$, $BD=22$, find the area of quadrilateral $ABCD$.



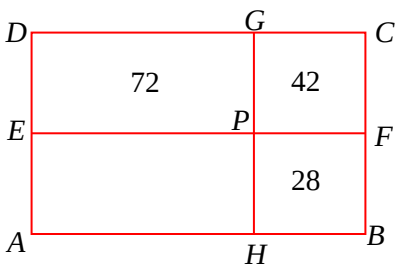
19. 将 4 支一样的蓝笔与 3 支一样的红笔排成一行，其中 3 支红笔必须相邻，有几种排法?
How many ways are there to arrange 4 identical blue pens and 3 identical red pens in a row if the 3 red pens must be next to each other?

20. 考完试后，老师计算班上学生的平均分数，得平均分数为 70 分。后来发现少算了一位考 87 分的学生李大卫的成绩。重新计算及确认后得全班的平均分数是 71 分。问这班上 (包括李大卫) 有多少位学生?
After an examination, the teacher computed the average mark scored by her students in the class to be 70. Later, the teacher found out that she has left out David Lee who scored 87. After a second calculation and a thorough checking, it was found that the average mark scored by all the students in the class is 71. How many students are there in this class (including David Lee)?

第 21 至第 25 题，问答题，每题 6 分。
Question 21 to Question 25, short questions, each question carries 6 marks.

21. 一班里有 27 位学生，编号 1 到 27。老师给每一位学生发铅笔，1 号学生发 1 支，2 号学生发 3 支，3 号学生发 5 支，4 号学生发 7 支，依此类推。问老师一共发了多少支铅笔?
There are 27 students in a class, numbered 1 to 27. A teacher distributes pencils to the students. Student no. 1 is given 1 pencil, student no. 2 is given 3 pencils, student no. 3 is given 5 pencils, student no. 4 is given 7 pencils, and so on. What is the total number of pencils distributed by the teacher?

22. 如下图所示，长方形 $DEPG$，长方形 $GPFC$ 及长方形 $PHBF$ 的面积分别为 72，42 及 28。求长方形 $ABCD$ 的面积。
As shown in the figure below, the rectangles $DEPG$, $GPFC$ and $PHBF$ have areas 72, 42 and 28 respectively. Find the area of rectangle $ABCD$.



23. 若 S 为 2018 的所有正的因数之和，求 $\left\lfloor \frac{S}{10} \right\rfloor$。

If S is the sum of all the positive factors of 2018, find $\left\lfloor \frac{S}{10} \right\rfloor$.

24. 已知 $\frac{7a-2b}{2a-b}=\frac{28}{5}$，求 $\frac{14a^2+24b^2}{5ab}$ 的值。

Given that $\frac{7a-2b}{2a-b}=\frac{28}{5}$, find the value of $\frac{14a^2+24b^2}{5ab}$.

25. 下图所示是一个有 39 行及 39 列的方格纸。在每一个方格内填入一个等于行数与列数之和的数，例如在第 3 行及第 4 列填入的数为 $3+4=7$。在所有的 39×39 个方格中，有多少个所填入的是奇数?

The figure below shows a grid paper with 39 rows and 39 columns. In each box, a number which is equal to the sum of the row number and the column number is filled. For example, the number filled in the 3rd row and the 4th column is $3+4=7$. Among all 39×39 boxes, how many of the numbers filled are odd numbers?

2	3	4	5		40
3	4	5	6		41
4	5	6	7		42
5	6	7	8		43
40	41	42	43		78

第 26 至第 30 题，问答题，每题 8 分。

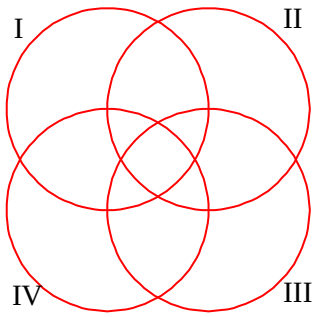
Question 26 to Question 30, short questions, each question carries 8 marks.

26. 有一群学生，其中有 48 人会弹钢琴，28 人会弹吉他，13 人两者都不会。问这群学生至少有多少人?

In a group of students, 48 of them play piano, 28 of them play guitar, 13 of them do not play any of the two instruments. What is the minimum number of students in this group?

27. 如下图所示，四个圆相交而分割形成 13 个区域，每个圆包含 7 个区域。将由 3 到 15 的 13 个整数填入这 13 个区域中，每个区域一个数，且没有两个区域填入相同的数。设 S_1，S_2，S_3 及 S_4 分别为 I，II，III 及 IV 各圆所包含区域中的数之和，且设 $S = S_1 + S_2 + S_3 + S_4$，求 S 的最大可能值。

As shown in the figure below, four circles intersect each other and form 13 regions, with each circle containing 7 regions. Thirteen integers, 3 to 15, are to be filled into these 13 regions, one number for each region, and no two regions sharing the same number. Let S_1, S_2, S_3 and S_4 be the sum of the numbers in the regions enclosed by the circles I, II, III and IV respectively, and let $S = S_1 + S_2 + S_3 + S_4$. Find the largest possible value of S.



28. 已知 340，532 及 628 三数除以 n 时的余数相等，求 n 的最大可能值。

Given that when the three numbers 340, 532 and 628 are divided by n, their remainders are the same. Find the largest possible value of n.

29. 张老师将巧克力分给 x 位男生及 y 位女生，男生每人 4 粒，女生每人 5 粒，一共分出 1231 粒。问 (x, y) 有多少组不同的可能值?

Ms Zhang gives chocolates to x boys and y girls. Each boy gets 4 chocolates and each girl gets 5. She gives away a total of 1231 chocolates. How many different possible combinations of (x, y) are there?

30. 有多少组正整数 (a, b, c) 满足 $a \le b \le c$ 且 $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$?

How many triples of positive integers (a, b, c) are there that satisfy $a \le b \le c$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$?