厦门大学马来西亚分校

陈景润杯中学数学比赛

CHEN JINGRUN'S CUP SECONDARY SCHOOL MATHEMATICS COMPETITION 2019

** 初阶组 **

JUNIOR CATEGORY

日期：2019 年 4 月 9 日　　　　时间：上午 10 时至中午 12 时

Date: 9th April 2019　　　　Time: 10:00 a.m. to 12:00 p.m.

考生须知

Instructions and Information

1. 本试卷共有 30 题。
   This paper contains 30 questions.
   - 第 1 题至第 10 题，选择题， 每题 4 分。
     Question 1 to Question 10, multiple choice questions, each question carries 4 marks.
   - 第 11 题至第 30 题，问答题，每题的答案是一个介于 0 至 1000 之间的整数。
     Question 11 to Question 30, short questions. For each question, the answer is an integer between 0 and 1000.
     - ➢ 第 11 题至第 20 题每题 5 分。
       Question 11 to Question 20, each question carries 5 marks.
     - ➢ 第 21 题至第 25 题每题 6 分。
       Question 21 to Question 25, each question carries 6 marks.
     - ➢ 第 26 题至第 30 题每题 8 分。
       Question 26 to Question 30, each question carries 8 marks.
2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题，必须填写 A，B，C，D 或 E 作为答案。每题只能填入一个答案，否则以答错论。
   Please use 2B pencils to write your answers in the appropriate boxes provided on the answer sheet. For each multiple choice question, please write A, B, C, D or E as answer. If more than one answer is found for a question, no credits would be given for that question.
3. 所有的图形并没有按照比例作图，只作为辅助之用。
   All the diagrams are not drawn to scale. They are intended as aids only.
4. 不许使用计算器，数学工具，手机或其他计算器。
   No calculators, maths stencils, mobile phones or other calculating aids are permitted.
5. 在答案纸上清楚写上姓名，考生编号，学校名称及就读年级。
   Write your name, candidate number, name of school and year of study clearly on the answer sheet.
6. 在监考老师宣布比赛开始之后，才可以翻开此考卷开始作答。
   Do not open this question booklet until you are told to do so.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中，$\lfloor x \rfloor$表示小于或等于 x 的最大整数。

例如：$\lfloor 2 \rfloor = 2$，$\lfloor -2 \rfloor = -2$，$\lfloor 2.6 \rfloor = 2$，$\lfloor -2.6 \rfloor = -3$。

In this paper, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题，选择题，每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 一支红笔的价格是 RM1.70。若林老师有RM50，她最多可以买到多少支红笔？
   The cost of a red pen is RM1.70. If Ms. Lim has RM50, at most how many red pens can she purchase?

   A. 25　　B. 26　　C. 27　　D. 28　　E. 29

   If Ms. Lim can purchase x red pens,

   $$1.7x \le 50$$
   $$x \le \frac{500}{17} = 29\frac{7}{17}$$

   Therefore, Ms. Lim can purchase at most 29 red pens.

   ANSWER:【E】

2. 求$\left\lfloor -\frac{5}{2} \right\rfloor + \lfloor -2 \rfloor + \left\lfloor -\frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \lfloor 2 \rfloor + \left\lfloor \frac{5}{2} \right\rfloor$。
   Find $\left\lfloor -\frac{5}{2} \right\rfloor + \lfloor -2 \rfloor + \left\lfloor -\frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \lfloor 2 \rfloor + \left\lfloor \frac{5}{2} \right\rfloor$.

   A. 0　　B. −1　　C. −2　　D. −3　　E. 1

   $$\left\lfloor -\frac{5}{2} \right\rfloor + \lfloor -2 \rfloor + \left\lfloor -\frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \lfloor 2 \rfloor + \left\lfloor \frac{5}{2} \right\rfloor = -3 - 2 - 1 + 0 + 2 + 2 = -2$$

   ANSWER:【C】

3. 一项工作，由 21 位工人做，60 天能完成。若由 28 位工人来做，需要几天来完成?
A task would take 60 days to complete by 21 workers. How many days does it take to complete by 28 workers?

A. 42 B. 45 C. 48 D. 75 E. 80

The number of days needed $= \frac{21\times 60}{28} = 45$

ANSWER:【B】

4. 在 2018 年，英华的月薪比英美的多 RM 83。在 2019 年，英华与英美都获得了 7% 的加薪后，英华的月薪比英美的月薪多 RM x。求最靠近 x 的整数。
In 2018, the monthly salary of Yinghua is RM 83 more than the monthly salary of Yingmei. In 2019, after both Yinghua and Yingmei receive a salary increment of 7%, the monthly salary of Yinghua is RM x more than the monthly salary of Yingmei. Find the integer closest to x.

A. 87 B. 88 C. 89 D. 90 E. 91

Let y be the monthly salary of Yinghua, and z be the monthly salary of Yingmei in 2018.
Then $y - z = 83$.
In 2019, the monthly salary of Yinghua is more than the monthly salary of Yingmei by

$$\begin{aligned} x &= 1.07y - 1.07z \\ &= 1.07\times 83 \\ &= 88.81 \end{aligned}$$

Hence, the integer closest to x is 89.

ANSWER:【C】

5. 工厂里有 67 堆书，每堆有 128 本。一工人要将这些书本包装在箱子中，每个箱子装 25 本。问包装后会剩下几本书?
There are 67 piles of books in a factory with each pile containing 128 books. A worker is asked to pack these books into boxes of 25 each. How many books would remain after the packing?

A. 1 B. 6 C. 11 D. 16 E. 21

$$\begin{aligned} 67\times128 &\equiv 17\times3 \quad \text{mod } 25 \\ &\equiv 1 \quad \text{mod } 25 \end{aligned}$$

So there will be 1 book remains.

ANSWER:【A】

6. 已知 P，Q，R，S，T 五个人的平均体重是 56 kg，P，Q，R 三人的平均体重是 58 kg，R，S，T 三人的平均体重是 57 kg，求 R 的体重。
Given that the average weight of the five persons P, Q, R, S, T is 56 kg, the average weight of three persons P, Q, R is 58 kg, and the average weight of three persons R, S, T is 57 kg, find the weight of R.

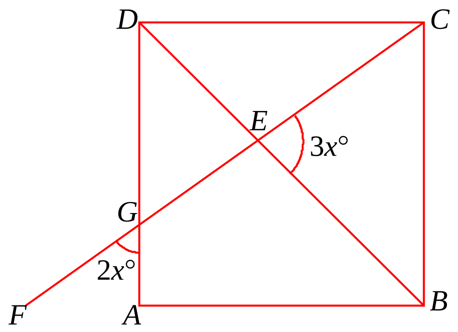
A. 65 kg B. 63 kg C. 61 kg D. 60 kg E. 59 kg

$$w_A + w_B + w_C + w_D + w_E = 5\times56 \quad ---(1)$$
$$w_A + w_B + w_C = 3\times58 \quad ---(2)$$
$$w_C + w_D + w_E = 3\times57 \quad ---(3)$$
$$(2)+(3)-(1): \quad w_C = 3\times58 + 3\times57 - 5\times56 = 56+6+3 = 65$$

ANSWER:【A】

7. 下图中，$ABCD$ 是正方形。若 $\angle BEC = 3x°$，$\angle AGF = 2x°$，求 x。

In the figure below, $ABCD$ is a square. If $\angle BEC = 3x°$, $\angle AGF = 2x°$, find x.



A. 25 B. 27 C. 31 D. 33 E. 35

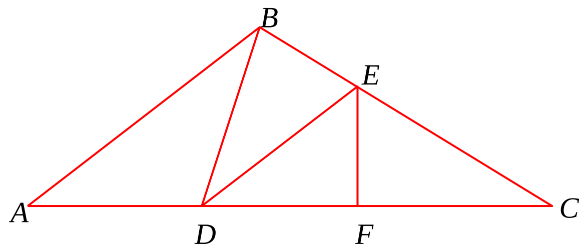
$\angle EDG = 45°$, $\angle DEG = 3x°$, $\angle DGE = 2x°$.

$$45 + 3x + 2x = 180$$
$$x = 27$$

ANSWER: 【B】

8. 下图中，$AB // DE$，$EF \perp AC$，DE 平分 $\angle BDC$。若 $\angle DBC = 71°$，$\angle FEC = 59°$，求 $\angle ABD$。

In the figure below, $AB // DE$, $EF \perp AC$, DE bisects $\angle BDC$. If $\angle DBC = 71°$, $\angle FEC = 59°$, find $\angle ABD$.



A. 31° B. 33° C. 35° D. 37° E. 39°

Let $\angle BAC = x$. Then $\angle BDE = \angle EDF = x$.

$$\angle ECF = 90° - 59° = 31°$$

In ΔBDC, $2x + 31° + 71° = 180°$

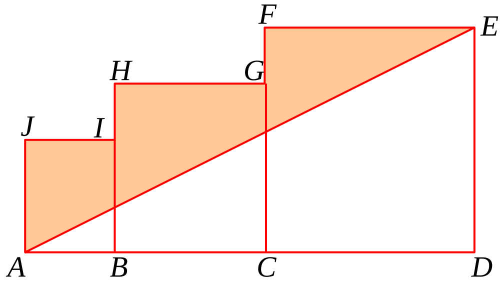
Hence, $x = 39°$

$\angle ABD = \angle BDE = x = 39°$

ANSWER: 【E】

9. 下图中， $ABIJ$ ， $BCGH$ ， $CDEF$ 是长方形， $AB=3$ ， $BI=4$ ， $BC=5$ ， $CG=6$ ， $CD=7$ ， $DE=8$ 。求阴影部分的面积。

In the figure below, $ABIJ$, $BCGH$, $CDEF$ are rectangles. Given that $AB=3$, $BI=4$, $BC=5$, $CG=6$, $CD=7$, $DE=8$, find the area of the shaded region.



A. 32 B. 34 C. 36 D. 38 E. 40

$AD=3+5+7=15$

The area of the shaded region is $3\times4+5\times6+7\times8-\frac{1}{2}\times15\times8=38$.

ANSWER:【D】

10. 在 2008 年，明明与亮亮的岁数之比是$3:4$。在 2020 年，明明与亮亮的岁数之比将会是$5:6$。在哪一年，明明与亮亮的岁数之比会是$6:7$？

In 2008, the ratio of the age of Mingming to the age of Liangliang was $3:4$. In 2020, the ratio of the age of Mingming to the age of Liangliang would be $5:6$. In which year would the ratio of the age of Mingming to the age of Liangliang be $6:7$?

A. 2023 B. 2024 C. 2025 D. 2026 E. 2027

Let the ages of Mingming and Liangliang be $3k$ and $4k$ respectively in year 2008.
Then in 2020,

$$\frac{3k+12}{4k+12}=\frac{5}{6}$$
$$18k+72=20k+60$$
$$k=6$$

If $(3k+a):(4k+a)=6:7$,

$$\frac{3k+a}{4k+a}=\frac{6}{7}$$
$$21k+7a=24k+6a$$
$$a=3k$$
$$=18$$

This implies that the ratio of the age of Mingming to the age of Liangliang would be $6:7$ in 2026.

ANSWER:【D】

第 11 至第 20 题，问答题，每题 5 分。
Question 11 to Question 20, short questions, each question carries 5 marks.

11. 已知$a+b=31$， $ab=100$，求$(a-b)^2$。

Given that $a+b=31$, $ab=100$, find $(a-b)^2$.

$$(a-b)^2=(a+b)^2-4ab$$
$$=31^2-4\times100$$
$$=561$$

ANSWER:【561】

12. 已知 x 是一正数使得 $\frac{21}{47}=\cfrac{1}{2+\cfrac{1}{x+\cfrac{1}{x+1}}}$，求 $\lfloor x \rfloor$。

Given that x is a positive number such that $\frac{21}{47}=\cfrac{1}{2+\cfrac{1}{x+\cfrac{1}{x+1}}}$, find $\lfloor x \rfloor$.

$$2+\cfrac{1}{x+\cfrac{1}{x+1}}=\frac{47}{21}$$

$$\cfrac{1}{x+\cfrac{1}{x+1}}=\frac{5}{21}$$

$$x+\frac{1}{x+1}=\frac{21}{5}$$

$$5x^2-16x-16=0$$

$$(5x+4)(x-4)=0$$

$$\because\ x>0, \quad \therefore\ x=4, \quad \lfloor x \rfloor=4$$

ANSWER:【4】

13. 买 5 支铅笔与 7 本练习簿要 RM15.65，买 7 支铅笔与 5 本练习簿要 RM15.43。买 100 支铅笔要多少令吉?

It costs RM15.65 to buy 5 pencils and 7 exercise books. It costs RM15.43 to buy 7 pencils and 5 exercise books. How much does it cost in RM to buy 100 pencils?

Assume that it costs RM x to buy 100 pencils, and it costs RM y to buy 100 exercise books. Then

$$\begin{cases} 5x+7y=1565 \quad ------(1) \\ 7x+5y=1543 \quad ------(2) \end{cases}$$

$\frac{(1)-(2)}{2}$: $\quad 2(y-x)=22$

$$y-x=11 \quad ------(3)$$

$\frac{(1)+(2)}{12}$: $\quad 12(x+y)=3108$

$$x+y=259 \quad ------(4)$$

$\frac{(4)-(3)}{2}$: $\quad x=124$

ANSWER:【124】

14. 求 $\sqrt{240\times244+2440+9}$。

Find $\sqrt{240\times244+2440+9}$.

Let $x=244$. Then

$$\begin{aligned} \sqrt{240\times244+2440+9} &= \sqrt{x(x-4)+10x+9} \\ &= \sqrt{x^2+6x+9} \\ &= \sqrt{(x+3)^2} \\ &= x+3 \\ &= 247 \end{aligned}$$

ANSWER:【247】

15. 已知 19 个连续奇数的和为 2185。求这 19 个数中最小的数。

Given that the sum of 19 consecutive odd numbers is 2185. Find the smallest of these 19 numbers.

Let these 19 numbers be $a-18$, $a-16$, $\cdots$, $a-2$, a, $a+2$, $\cdots$, $a+16$, $a+18$.
Their sum is $19a=2185$.
Hence, $a=115$
The smallest number is $115-18=97$.

ANSWER:【97】

16. 已知 $N=1\times2\times3\times\cdots\times500$ 是由 1 到 500 的整数的乘积。若 p 是可以整除 N 的质数，求 p 的最大可能值。

Given that $N=1\times2\times3\times\cdots\times500$ is the product of the integers from 1 to 500. If p is a prime number that divides N, find the largest possible value of p.

p is the largest prime less than or equal to 500. It is easy to verify that 499 is a prime number. Hence, the largest possible value of p is 499.

ANSWER:【499】

17. 已知 $\overline{abc}$ 是一三位数且 $81a+9b+c=636$，求三位数 $\overline{abc}$。

Given that $\overline{abc}$ is a three-digit number and $81a+9b+c=636$. Find the three-digit number $\overline{abc}$.

$9(9a+b)=636-c$

Hence, c is a positive integer between 0 and 9 such that $636-c$ is divisible by 9. Since 636 leaves a remainder of 6 when it is divided by 9, we find that $c=6$, and

$$9a+b=70$$

Similarly, b is a positive integer between 0 and 9 such that $70-b$ is divisible by 9. Since 70 leaves a remainder of 7 when it is divided by 9, we find that $b=7$, and hence $a=7$.
The three-digit number $\overline{abc}$ is 776.

ANSWER:【776】

18. 若 x 是整数，求 $|x+225|+|x+404|$ 的最小可能值。

If x is an integer, find the smallest possible value of $|x+225|+|x+404|$.

If $x<-404$,

$$\begin{aligned}|x+225|+|x+404| &= -(x+225)-(x+404)\\ &> 2\times 404-404-225\\ &=179\end{aligned}$$

If $x>-225$,

$$\begin{aligned}|x+225|+|x+404| &= (x+225)+(x+404)\\ &> -2\times 225+404+225\\ &=179\end{aligned}$$

If $-404\le x\le -225$,

$$\begin{aligned}|x+225|+|x+404| &= -(x+225)+(x+404)\\ &=179\end{aligned}$$

Hence, the smallest possible value of $|x+225|+|x+404|$ is 179.

ANSWER:【179】

19. 三位老师 A，B，C 各有 x 粒巧克力。老师 A 将她的巧克力平分给 12 位学生，老师 B 将她的平分给 21 位学生，而老师 C 将她的平分给 56 位学生。求 x 的最小可能值。

[注： x 是正整数]

Each of the three teachers A, B and C has x chocolates. Teacher A distributes her chocolates evenly to 12 students, teacher B distributes hers evenly to 21 students, and teacher C distributes hers evenly to 56 students. Find the smallest possible value of x.

[Note: x is a positive integer.]

x is the least common multiple of 12, 21 and 56

$12=2^2\times 3$

$21=3\times 7$

$56=2^3\times 7$

Hence, $x=2^3\times 3\times 7=168$.

ANSWER:【168】

20. 在下午 2 时 x 分，时钟上的时针与分针成 $180°$ 角。若 $x=\frac{a}{b}$，其中 a，b 是互质的正整数，求 $a+b$ 的值。

At x minutes after 2 o’clock in the afternoon (here x is less than 60), the hour-hand and the minute-hand on a clock form an angle of $180°$. If $x=\frac{a}{b}$, where a and b are relatively prime positive integers, find the value of $a+b$.

At 2:00 p.m., the hour hand and the minute hand form an angle of $\frac{2}{12}\times 360°=60°$.

From 2 o’clock to x minutes after 2 o’clock, the minute hand has moved an angle of $\frac{x}{60}\times 360°=6x°$; while the hour hand has moved an angle of $\frac{6x°}{12}=\frac{x°}{2}$.

Hence,

$$6x°-60°-\frac{x°}{2}=180°$$
$$\frac{11x°}{2}=240°$$
$$x=\frac{480}{11}$$

Hence, $a=480$, $b=11$, $a+b=491$.

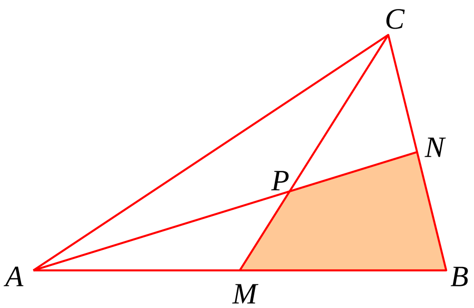
ANSWER:【491】

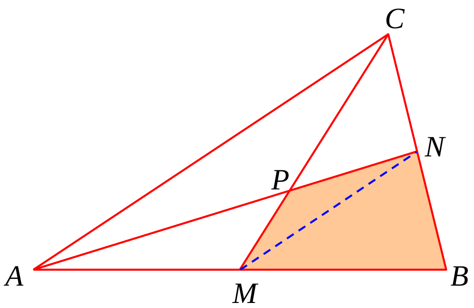
第 21 至第 25 题，问答题，每题 6 分。

Question 21 to Question 25, short questions, each question carries 6 marks.

21. 下图中，M 及 N 分别为线段 AB 及 BC 的中点。若 ΔABC 的面积为 S_1，四边形 $BMPN$ 的面积为 S_2，求 $\frac{210S_2}{S_1}$。

In the figure below, M and N are respectively the midpoints of the line segments AB and BC. If the area of ΔABC is S_1, and the area of the quadrilateral $BMPN$ is S_2, find $\frac{210S_2}{S_1}$.





$\frac{BM}{BA}=\frac{BN}{BC}$, $\therefore$ $MN//AC$, $\Delta MNP \sim \Delta CAP$, $\frac{MP}{CP}=\frac{MN}{AC}=\frac{BM}{BA}=\frac{1}{2}$

$\frac{BM}{BA}=\frac{1}{2}$, $\therefore$ $S_{\Delta BMC}=\frac{1}{2}S_{\Delta ABC}$

$\frac{BN}{BC}=\frac{1}{2}$, $\therefore$ $S_{\Delta BMN}=\frac{1}{2}S_{\Delta BMC}=\frac{1}{4}S_{\Delta ABC}$, $S_{\Delta CMN}=S_{\Delta BMN}=\frac{1}{4}S_{\Delta ABC}$

$\frac{MP}{MC}=\frac{1}{3}$, $\therefore$ $S_{\Delta MPN}=\frac{1}{3}S_{\Delta CMN}=\frac{1}{12}S_{\Delta ABC}$

Hence, the area of quadrilateral $BMPN$ is

$$S_2=S_{\Delta BMN}+S_{\Delta MPN}=\left(\frac{1}{4}+\frac{1}{12}\right)S_1=\frac{1}{3}S_1$$

Therefore, $\frac{210S_2}{S_1}=70$

ANSWER:【70】

22. 求 $\frac{1}{\sqrt{9}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{13}}+\frac{1}{\sqrt{13}+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{17}}+\cdots+\frac{1}{\sqrt{959}+\sqrt{961}}$ 。

Find $\frac{1}{\sqrt{9}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{13}}+\frac{1}{\sqrt{13}+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{17}}+\cdots+\frac{1}{\sqrt{959}+\sqrt{961}}$.

$$\frac{1}{\sqrt{9}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{13}}+\frac{1}{\sqrt{13}+\sqrt{15}}+\frac{1}{\sqrt{15}+\sqrt{17}}+\cdots+\frac{1}{\sqrt{959}+\sqrt{961}}$$
$$=\frac{\sqrt{11}-\sqrt{9}}{2}+\frac{\sqrt{13}-\sqrt{11}}{2}+\frac{\sqrt{15}-\sqrt{13}}{2}+\cdots+\frac{\sqrt{961}-\sqrt{959}}{2}$$
$$=\frac{\sqrt{961}-\sqrt{9}}{2}$$
$$=\frac{31-3}{2}$$
$$=14$$

ANSWER:【14】

23. 从 1 开始，将整数分组如下：

第一组：[1]

第二组：[2，3，4]

第三组：[5，6，7，8，9]

第四组：[10，11，12，13，14，15，16]

⋮

其中第一组有 1 个数，第二组 3 个，第三组 5 个，依此类推，每一组比前一组多两个数。求 2019 所在的那一组有几个数。

The integers beginning from 1 are grouped as follows:

1^{st} group: [1]

2^{nd} group: [2, 3, 4]

3^{rd} group: [5, 6, 7, 8, 9]

4^{th} group: [10, 11, 12, 13, 14, 15, 16]

⋮

There is 1 number in the first group, 3 in the second, 5 in the third, and so on. Each group has two additional numbers than the previous group. How many numbers are there in the group that contains the number 2019?

The n^{th} group contains $(2n-1)$ numbers from $(n-1)^2+1$ to n^2.

If 2019 is in the n^{th} group

$$(n-1)^2 < 2019 \le n^2$$

Since $44^2=1936$, $45^2=2025$, $n=45$, and there are 89 numbers in the group containing 2019.

ANSWER:【89】

24. 小于 2100 的正整数中，有多少个与 105 互质?

Among the positive integers that are less than 2100, how many of them are relatively prime to 105?

$105 = 3 \times 5 \times 7$

An integer is relatively prime to 105 if and only if it is not divisible by 3, 5 or 7.

Among the positive integers that are less than 2100, $\frac{2100}{3}$ is divisible by 3;

$\frac{2100}{5}$ is divisible by 5;

$\frac{2100}{7}$ is divisible by 7;

$\frac{2100}{3\times5}$ is divisible by 3 and 5;

$\frac{2100}{5\times7}$ is divisible by 5 and 7;

$\frac{2100}{3\times7}$ is divisible by 3 and 7;

$\frac{2100}{3\times5\times7}$ is divisible by 3, 5 and 7;

Hence, among the positive integers that are less than 2100, the number of those relatively prime to 105 is

$$\begin{aligned}
&2100-\frac{2100}{3}-\frac{2100}{5}-\frac{2100}{7}+\frac{2100}{3\times5}+\frac{2100}{5\times7}+\frac{2100}{3\times7}-\frac{2100}{3\times5\times7}\\
&=2100\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)\\
&=2100\times\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}\\
&=20\times48\\
&=960
\end{aligned}$$

ANSWER:【960】

25. 已知a，b，c是实数且$(b-196a)^2-128(b-14c)(c-14a)=0$，求$\dfrac{b-14c}{c-14a}$的最大可能值。

Given that a, b, c are real numbers and $(b-196a)^2-128(b-14c)(c-14a)=0$, find the largest possible value of $\dfrac{b-14c}{c-14a}$.

Let $u=b-14c$, $v=c-14a$. Then $u+14v=b-14c+14c-196a=b-196a$

Hence,

$$(u+14v)^2-128uv=0$$
$$u^2-100uv+196v^2=0$$
$$(u-2v)(u-98v)=0$$
$$u=2v \quad \text{or} \quad u=98v$$

This implies that the largest possible value of $\dfrac{b-14c}{c-14a}=\dfrac{u}{v}$ is 98.

ANSWER:【98】

第 26 至第 30 题，问答题，每题 8 分。

Question 26 to Question 30, short questions, each question carries 8 marks.

26. 有 93 粒球，编号1至 93。现将这 93 粒球分给k位学生，使得每人所获得的球的号码之和一样，求k的最大可能值。[注：如果某位学生只获得一粒球，则他所获得的球的号码之和为该粒球的号码。]

There are 93 balls, numbered 1 to 93. Now these balls are distributed to k students in such a way that the sum of the numbers on the balls obtained by each student is the same. Find the largest possible value of k. [Note: If a student gets only one ball, then the sum of the numbers on the balls he obtained is the number on that ball.]

We claim that the largest possible value of k is 47, where the first student gets balls numbered 1 and 92, the second student gets balls numbered 2 and 91, the third student gets balls numbered 3 and 90, and so on. The 46th student gets balls numbered 46 and 47, and the 47th student get the ball numbered 93. The sum of the numbers on the balls obtained by each student is 93.

k could not be larger than 47, otherwise, there will be at least two students getting only one ball. But then, these two students will have a different sum.

ANSWER:【47】

27. 将 13 颗一样的糖果分给 3 位小孩，每人至少一颗，有多少种不同的方法?
How many ways are there to distribute 13 identical sweets to 3 children so that each of them will get at least 1 sweet?

Since the sweets are identical, the only thing that matters is the number of sweets each child obtains. Let x_1, x_2, x_3 be respectively the number of sweets obtained by each of the three children. Listing down the possible values of x_1, x_2, x_3 in a systematic way:

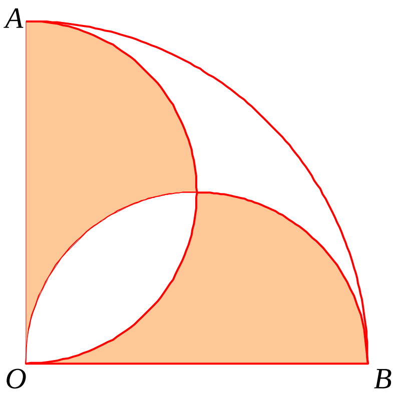
x_1	x_2	x_3
1	1	11
1	2	10
⋮		⋮
1	11	1
2	1	10
2	2	9
⋮		⋮
2	10	1
⋮		⋮
10	1	2
10	2	1
11	1	1

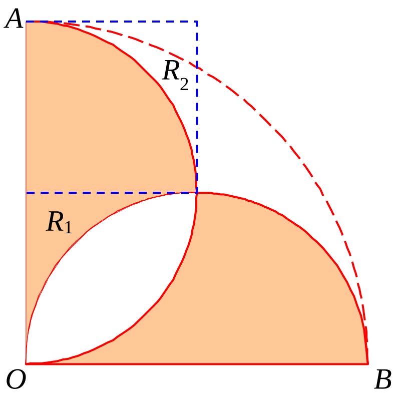
We find that the number of ways to distribute the sweets is
$11+10+9+8+7+6+5+4+3+2+1=66$

ANSWER:【66】

28. 下图所示是一四分之一圆及两个半圆。若 $OA = 38$，阴影部分的面积为 S，求 $\lfloor S \rfloor$。

The figure below shows a quarter-circle and two semi-circles. If $OA = 38$ and the area of the shaded region is S, find $\lfloor S \rfloor$.





Notice that the area of R_1 is the same as the area of R_2. Hence,

$$S = 2 \times 19^2 = 722$$

ANSWER:【722】

29. 已知 x 是小于 1000 的正整数，且 x^2 除以 341 余 9，求 x 的最大可能值。

Given that x is a positive integer less than 1000, and x^2 leaves a remainder of 9 when it is divided by 341. Find the largest possible value of x.

$x^2-9=(x+3)(x-3)$ is divisible by $341=11\times 31$, therefore it is divisible by 11 and 31.

So, either $x+3$ or $x-3$ must be divisible by 11, and either $x+3$ or $x-3$ must be divisible by 31. Thus, x must satisfy one of the following systems:

$$\begin{cases} x\equiv 3 \mod 11 \\ x\equiv 3 \mod 31 \end{cases} \text{ or } \begin{cases} x\equiv -3 \mod 11 \\ x\equiv 3 \mod 31 \end{cases} \text{ or } \begin{cases} x\equiv 3 \mod 11 \\ x\equiv -3 \mod 31 \end{cases} \text{ or } \begin{cases} x\equiv -3 \mod 11 \\ x\equiv -3 \mod 31 \end{cases}$$

Hence, $x\equiv 3 \mod 341$

or $x\equiv 96 \mod 341$

or $x\equiv 245 \mod 341$

or $x\equiv 338 \mod 341$

The largest possible value of x is $341\times 2+245=927$.

ANSWER:【927】

30. 有多少组正整数(x, y, z)满足等式 $x+y+z+xy+yz+zx+xyz=2019$?

How many triples of positive integers (x, y, z) satisfy the following equality
$x+y+z+xy+yz+zx+xyz=2019$?

$$x+y+z+xy+yz+zx+xyz=2019$$
$$1+x+y+z+xy+yz+zx+xyz=2020$$
$$(1+x)(1+y)(1+z)=2^2\times5\times101$$
$$1+x\geq2,\ 1+y\geq2,\ 1+z\geq2$$

Assume first that $x\leq y\leq z$. Then

$$\begin{cases}1+x=2\\1+y=2\\1+z=505\end{cases},\ \begin{cases}1+x=2\\1+y=10\\1+z=101\end{cases},\ \begin{cases}1+x=2\\1+y=5\\1+z=202\end{cases},\ \begin{cases}1+x=4\\1+y=5\\1+z=101\end{cases}$$

From these, we find that (x, y, z) can be

$(1,1,504)$, $(1,504,1)$, $(504,1,1)$

$(1,9,100)$, $(9,1,100)$, $(9,100,1)$, $(1,100,9)$, $(100,1,9)$, $(100,9,1)$

$(1,4,201)$, and five other permutations

$(3,4,100)$, and five other permutations

Hence, there are altogether $3+6+6+6=21$ triples of (x, y, z).

ANSWER:【21】