



厦门大学马来西亚分校  
陈景润杯中学数学比赛



**CHEN JINGRUN'S CUP SECONDARY SCHOOL  
MATHEMATICS COMPETITION 2020**

**JUNIOR CATEGORY**

**\*\* 初阶组 \*\***

Date: 22<sup>nd</sup> August 2020

日期: 2020年8月22日

Time: 2:00 pm to 4:30 pm

时间: 下午2时至下午4时半

**Instructions and Information**

**考生须知**

1. This paper contains 30 questions. Each question is a short question which carries 4 to 8 points.

本试卷有30道题目，每题都是问答题，分数介于4分至8分之间。

2. The answer for each question is an integer.

每题的答案是一个整数。

3. All the diagrams are not drawn to scale. They are intended as aids only.

所有的图形并没有按照比例作图，只作为辅助之用。

4. In this paper,  $\lfloor x \rfloor$  denotes the greatest integer less than or equal to  $x$ .

For example,  $\lfloor 2 \rfloor = 2$ ,  $\lfloor -2 \rfloor = -2$ ,  $\lfloor 2.6 \rfloor = 2$ ,  $\lfloor -2.6 \rfloor = -3$ .

在这份试卷中， $\lfloor x \rfloor$ 表示小于或等于 $x$ 的最大整数。

例如： $\lfloor 2 \rfloor = 2$ ， $\lfloor -2 \rfloor = -2$ ， $\lfloor 2.6 \rfloor = 2$ ， $\lfloor -2.6 \rfloor = -3$ 。

**Question J-01 [4 points]**

The 2020 students in a school are divided into 7 groups to compete in a game. Any two groups can have the same number of students. What is the minimum number of students in the largest group(s)?

一间学校有 2020 位学生。如果将这些学生分成 7 组来进行运动会，任意两组可以有相同的人数，那么人数最多的组至少有多少人？

**Question J-02 [4 points]**

There are 11 students that took part in a mathematics exam. By taking 10 students at a time and sum up their marks, the following 11 sums are obtained:

749 743 740 737 733 729 726 723 718 714 708

What is the lowest mark scored by these 11 students?

有 11 位学生参加数学考试。每次取 10 位学生的分数加起来，得到以下的 11 个和：

749 743 740 737 733 729 726 723 718 714 708

这 11 位学生分数最低的是几分？

**Question J-03 [4 points]**

Tianying Company has  $N$  employees,  $\frac{3}{8}$  of them work in the Johor branch, and 15% of them work in the Penang branch. The remaining 437 employees work in the headquarter in Kuala Lumpur. Find  $N$ .

天英公司有  $N$  位员工， $\frac{3}{8}$  的员工在柔佛分部，15% 的员工在檳城分部，剩下的 437 位员工在吉隆坡总部。求  $N$ 。

**Question J-04 [4 points]**

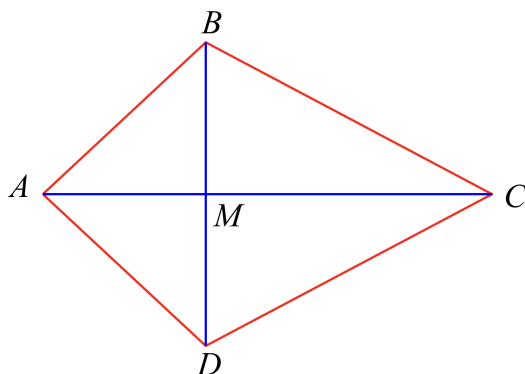
If  $m$  and  $n$  are positive integers such that  $n^5 = 18000m$ , find the smallest possible value of  $m$ .

已知  $m$  与  $n$  是正整数且  $n^5 = 18000m$ ，求  $m$  的最小可能值。

**Question J-05 [4 points]**

In the figure shown below,  $ABCD$  is a kite. Given that  $AM = 15$ ,  $CM = 48$ , and the area of the kite is 1260, find the perimeter of the kite.

下图中， $ABCD$  是风筝形。已知  $AM = 15$ ， $CM = 48$ ，风筝形的面积为 1260。求风筝形的周长。

**Question J-06 [4 points]**

There are two balls, one red and one yellow. The volume of the yellow ball is 72.8% more than the volume of the red ball, while the surface area is  $x\%$  more. Find  $x$ .

有两粒球，一粒红色，一粒黄色。黄球的体积比红球的大 72.8%，表面积比红球的大  $x\%$ ，求  $x$ 。

**Question J-07 [4 points]**

Given that  $x$  and  $y$  are real numbers such that  $\frac{11y - 6x}{17x + 8y} = \frac{294}{577}$ , find  $\frac{6x + 11y}{17x - 8y}$ .

已知  $x$  与  $y$  是实数且  $\frac{11y - 6x}{17x + 8y} = \frac{294}{577}$ , 求  $\frac{6x + 11y}{17x - 8y}$ 。

**Question J-08 [4 points]**

Given that all the two-digit positive integers are divided into 5 groups such that the sum of the numbers in each group is  $N$ . Find  $N$ .

将所有正的二位数分成 5 组, 使得每组的和都是  $N$ 。求  $N$ 。

**Question J-09 [4 points]**

If  $a$  and  $b$  are real numbers such that  $a - 23 \neq 0$ ,  $3b + 37 \neq 0$ ,  $a \neq 3b + 60$  and

$$\frac{2}{a - 23} - \frac{2}{3b + 37} = 3b - a + 60,$$

find the value of  $3ab + 37a - 69b$ .

已知  $a$  与  $b$  是实数。  $a - 23 \neq 0$ ,  $3b + 37 \neq 0$ ,  $a \neq 3b + 60$  且

$$\frac{2}{a - 23} - \frac{2}{3b + 37} = 3b - a + 60$$

求  $3ab + 37a - 69b$  的值。

**Question J-10 [4 points]**

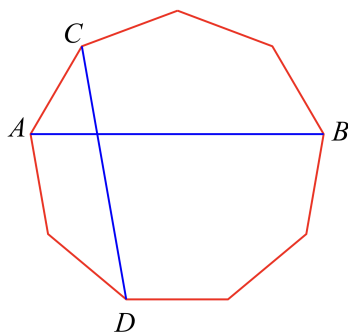
How many four-digit numbers leaves a remainder of 7 when divided by 13?

有多少个四位数除以 13 时余数为 7?

**Question J-11 [5 points]**

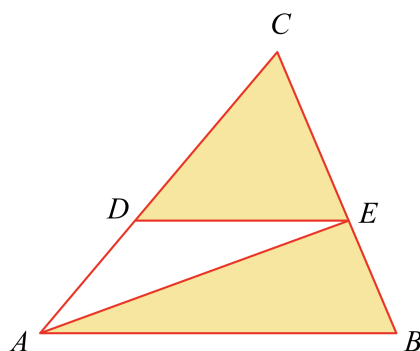
The figure shown below is a regular 9-gon. If the acute angle between the diagonals  $AB$  and  $CD$  is  $x^\circ$ , find  $x$ .

下图所示为一正九边形。若对角线  $AB$  与  $CD$  所夹的锐角为  $x^\circ$ ，求  $x$ 。

**Question J-12 [5 points]**

In the figure shown below,  $DE \parallel AB$ ,  $AD : DC = m : n$ , where  $m$  and  $n$  are relatively prime positive integers and  $m < n$ . If the area of the shaded region is  $\frac{403}{529}$  of the area of the whole triangle  $\triangle ABC$ , find the value of  $n$ .

下图中， $DE \parallel AB$ ， $AD : DC = m : n$ ，其中  $m$  及  $n$  是互质的正整数且  $m < n$ 。如果着色部分的面积占整个三角形  $\triangle ABC$  面积的  $\frac{403}{529}$ ，求  $n$  的值。



**Question J-13 [5 points]**

How many pairs of positive integers  $(x, y)$  satisfy the equation  $101x + 20y = 1010101$ ?

有多少对正整数  $(x, y)$  满足方程式  $101x + 20y = 1010101$  ?

**Question J-14 [5 points]**

Given that

$$\frac{xy}{11x + 12y} = \frac{110}{21}, \quad \frac{xy}{10x + 11y} = \frac{132}{23}.$$

Find the value of  $x + y$ .

已知

$$\frac{xy}{11x + 12y} = \frac{110}{21}, \quad \frac{xy}{10x + 11y} = \frac{132}{23}.$$

求  $x + y$  的值.

**Question J-15 [5 points]**

Given that

$$(1 + x) + (1 + 2x)^2 + (1 + 3x)^3 + (1 + 4x)^4 + (1 + 5x)^5 = a_0 + a_1x + a_2x^2 + \dots + a_{25}x^{25}.$$

Find  $a_2 + a_4 + a_6 + \dots + a_{24}$ .

已知

$$(1 + x) + (1 + 2x)^2 + (1 + 3x)^3 + (1 + 4x)^4 + (1 + 5x)^5 = a_0 + a_1x + a_2x^2 + \dots + a_{25}x^{25}$$

求  $a_2 + a_4 + a_6 + \dots + a_{24}$ 。

**Question J-16 [5 points]**

A school has four clubs whose members are students in this school. Each club has 99 members. Every two clubs have 33 common members. Every three clubs have 11 common members. There is exactly one student that joins all four clubs. At least how many students does this school have?

已知一间学校有 4 个学会，每个学会的会员都是该校的学生。每个学会会有 99 位会员，每两个学会会有 33 位共同会员，每三个学会会有 11 位共同会员，恰有一位学生是这四个学会的共同会员。这间学校最少有几位学生？

**Question J-17 [5 points]**

Find the sum of the real solutions to the equation

$$\frac{1}{x-12} + \frac{6}{x-14} = \frac{3}{x-17} + \frac{4}{x-11}.$$

求方程式

$$\frac{1}{x-12} + \frac{6}{x-14} = \frac{3}{x-17} + \frac{4}{x-11}$$

的所有实数解的和。

**Question J-18 [5 points]**

Let

$$a_n = \left\lfloor \frac{n}{13} \right\rfloor,$$

where  $\lfloor x \rfloor$  is the largest integer not larger than  $x$ . Find the sum

$$a_1 + a_2 + \dots + a_{200}.$$

设

$$a_n = \left\lfloor \frac{n}{13} \right\rfloor$$

其中  $\lfloor x \rfloor$  是不大于  $x$  的最大整数。求

$$a_1 + a_2 + \dots + a_{200}$$

**Question J-19 [5 points]**

Let

$$S = \{r \mid r \text{ is the remainder when } n^2 \text{ is divided by } 100, \text{ where } n \text{ is an integer}\}.$$

Find the sum of the elements in  $S$ .

设

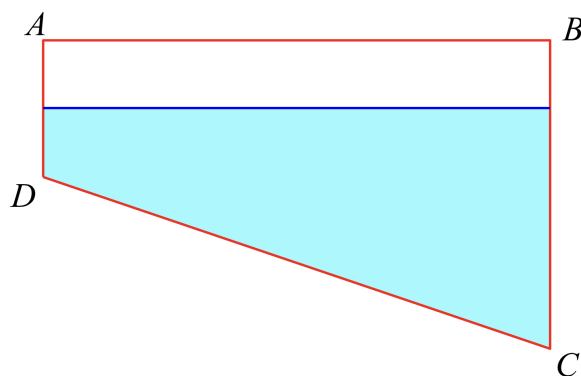
$$S = \{r \mid r \text{ 是 } n^2 \text{ 除以 } 100 \text{ 的余数, 其中 } n \text{ 是整数}\}$$

求  $S$  中元素的和。

**Question J-20 [5 points]**

The horizontal cross section of a swimming pool is a rectangle, while the vertical cross section is a trapezium. The figure below shows the vertical cross section of the swimming pool. The shallow side of the pool  $AD$  has height 100 cm, while the deeper side  $BC$  has height 180 cm. The pool is originally full of water. After the water level drops by  $k$  cm, the pool only left with 77% of water. Find the value of  $10k$ .

一泳池的水平面是一长方形，直切面是梯形。下图所示为泳池的直切面。泳池浅的一边  $AD$  的高度是 100 cm，深的一边  $BC$  的高度是 180 cm。开始时泳池的水是满的，水面下降了  $k$  cm 后，泳池剩下 77% 的水。求  $10k$  的值。



**Question J-21 [6 points]**

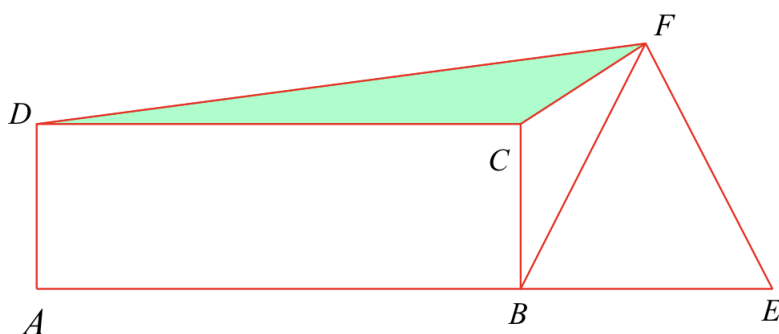
If the first 10 terms of the sequence of odd positive integers  $1, 3, 5, 7, \dots$  are merged, we obtain the 15-digit number 135791113151719. If the first  $n$  terms of the sequence are merged, we obtain a 2017-digit number  $N$ . Find the last three digits of  $N$ .

将正奇数数列  $1, 3, 5, 7, \dots$  的前面 10 项连在一起, 就得到 135791113151719 这个 15 位数。如果将前面  $n$  项连在一起会得到一个 2017 位数  $N$ , 求  $N$  的最后三位数。

**Question J-22 [6 points]**

In the figure shown below,  $ABE$  is a straight line.  $ABCD$  is a rectangle with  $AB = 74$  and  $AD = 34$ .  $\triangle BEF$  is an equilateral triangle.  $\triangle BCF$  is an isosceles triangle. Find the area of  $\triangle CDF$ .

下图中,  $ABE$  是一直线。  $ABCD$  是长方形,  $AB = 74$ ,  $AD = 34$ 。  $\triangle BEF$  是等边三角形,  $\triangle BCF$  是一等腰三角形。求  $\triangle CDF$  的面积。

**Question J-23 [6 points]**

Given that  $a, b, c$  are three positive integers. The greatest common divisor of  $a$  and  $b$  is 540, the greatest common divisor of  $a$  and  $c$  is 504, and the least common multiple of  $b$  and  $c$  is 83160. If  $m$  is the greatest common divisor of  $b$  and  $c$ , find the largest possible value of  $m$ .

已知  $a, b, c$  三数是正整数,  $a$  与  $b$  的最大公因数是 540,  $a$  与  $c$  的最大公因数是 504,  $b$  与  $c$  的最小公倍数是 83160。若  $m$  是  $b$  与  $c$  的最大公因数, 求  $m$  的最大可能值。

**Question J-24 [6 points]**

Let  $S = \{1, 3, 5, \dots, 99\}$  be the set containing all positive odd integers less than 100. For a set  $A$  that is a subset of  $S$ , define  $f(A)$  to be the sum of the elements in  $A$ . What is the maximum number of elements that  $A$  can have if  $f(A) = 1900$ ?

设  $S = \{1, 3, 5, \dots, 99\}$  为小于 100 的正奇数集合。对于  $S$  的子集合  $A$ ，定义  $f(A)$  为  $A$  中所有元素的和。若  $f(A) = 1900$ ， $A$  最多有几个元素？

**Question J-25 [6 points]**

Given that the degrees of the four internal angles of a quadrilateral are  $a, a + d, a + 2d, a + 3d$ , where  $a$  and  $d$  are positive integers. Find the largest possible value of  $d$ .

已知一个四边形的四个内角的度数分别为  $a, a + d, a + 2d, a + 3d$ ，其中  $a$  与  $d$  都是正整数。求  $d$  的最大可能值。

**Question J-26 [8 points]**

Given that a triangle is such that each of the three side lengths can only be one of the six integers

7 10 13 16 19 22

How many such triangles are there?

已知一三角形的三边长都必须是在以下六个数之中的一个：

7 10 13 16 19 22

有多少个这样的三角形？

**Question J-27 [8 points]**

A right-angled triangle is called perfect if the lengths of all three sides are integers, and one of the sides has length 100. How many perfect right-angled triangles are there?

如果一直角三角形三边的长都是整数，且其中一边的长等于100，我们称这三角形为完美三角形。有多少个直角三角形是完美的？

**Question J-28 [8 points]**

Let  $S$  be the set of positive integers less than 100. Given that any  $n$ -element subset of  $S$  contains two elements that are not relatively prime, find the smallest possible value of  $n$ .

设  $S$  是所有小于 100 的正整数的集合。已知  $S$  的任一个含有  $n$  个元素的子集合一定会有两个不互质的元素，求  $n$  的最小可能值。

**Question J-29 [8 points]**

Let

$$S = \{n \mid n \text{ is a positive integer less than } 100000, \text{ the product of digits of } n \text{ is } 180\}$$

Find the number of elements in  $S$ .

设

$$S = \{n \mid n \text{ 是小于 } 100000 \text{ 的正整数且 } n \text{ 的各位数字的乘积是 } 180\}$$

求  $S$  中元素的个数。

**Question J-30 [8 points]**

Given that  $a, b, x$  and  $y$  are real numbers such that

$$ax - by = 7$$

$$a^2x^3 - b^2y^3 = 22$$

$$a^3x^5 - b^3y^5 = 69$$

$$a^4x^7 - b^4y^7 = 216$$

$$a^5x^9 - b^5y^9 = K$$

Find  $K$ .

已知  $a, b, x$  及  $y$  是实数且

$$ax - by = 7$$

$$a^2x^3 - b^2y^3 = 22$$

$$a^3x^5 - b^3y^5 = 69$$

$$a^4x^7 - b^4y^7 = 216$$

$$a^5x^9 - b^5y^9 = K$$

求  $K$ 。