Question J-01 [5 points]

Today is Thursday. What is the day 2021 days later?

The code of the days are tabulated below. Enter the code as your answer.

今天是星期四。2021 天之后是星期几?

下表为各天的代码。请填入代码作为答案。

Day	天	Code /代码
Monday	星期一	1
Tuesday	星期二	2
Wednesday	星期三	3
Thursday	星期四	4
Friday	星期五	5
Saturday	星期六	6
Sunday	星期天	7

Answer: [2]

Solutions:

When 2021 is divided by 7, the remainder is 5. Hence, 2021 days later is Tuesday.

Question J-02 [5 points]

The average height of the two persons P and Q is 152 cm, while the average height of the five persons P, Q, R, S and T is 155 cm. What is the average height of the three persons R, S and T in cm?

P，Q 两人的平均身高是 152 cm，P，Q，R，S，T 五人的平均身高是 155 cm。R，S，T 三人的平均身高是多少 cm?

Answer: [157]

Solutions:

The total height of P and Q is 2×152 cm, the total height of P, Q, R, S, T is 5×155 cm. Hence, the average height of R, S and T is

$$\frac{5 \times 155 - 2 \times 152}{3} = 157\,\text{cm}$$

Question J-03 [5 points]

Xiaoming, Xiaolan, Wenying and Fenfen went to a stationery shop together. Xiaoming spent RM 3.20 to buy one pencil and one pen. Xiaolan spent RM 5.50 to buy one pencil and one exercise book. Wenying spent RM 4.30 to buy one pen and one ruler. If Fenfen bought 10 exercise books and 10 rulers, how much did she spend in RM?

小明，小兰，文英，芬芬四人一起去文具店。小明买一支铅笔和一支原子笔花了 RM 3.20，小兰买一支铅笔和一本作业簿花了 RM 5.50，文英买一支原子笔与一把尺花了 RM 4.30。若芬芬买了 10 本作业簿和 10 把尺，她花了多少令吉？

Answer: [66]

Solutions:

One pencil, one exercise book, one pen, one ruler cost RM 5.50 + RM 4.30 = RM 9.80.

Hence, one exercise book and one ruler cost RM 9.80 − RM 3.20 = RM 6.60.

10 exercise books and 10 rules cost RM 66.

Question J-04 [5 points]

How many positive factors does 120 have?

120 有几个正的因数?

Answer: [16]

Solutions:

$120 = 2^3 \times 3 \times 5$.

Any positive factor of 120 must be of the form $2^a \times 3^b \times 5^c$, where a can be 0, 1, 2 or 3, b can be 0 or 1, and c can also be 0 or 1. Hence, there are $4 \times 2 \times 2 = 16$ cases.

Question J-05 [5 points]

How many digits are there in the number $8^{10} \times 5^{26}$?

$8^{10} \times 5^{26}$ 这个数有几位数字?

Answer: [28]

Solutions:

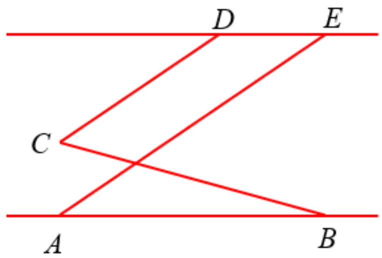
$$8^{10} \times 5^{26} = 2^{30} \times 5^{26} = 16 \times 10^{26}$$

Therefore, it has 28 digits.

Question J-06 [5 points]

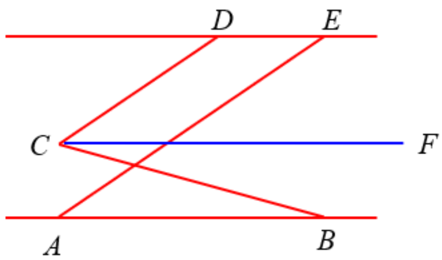
In the figure below, $AB//DE$, $CD//AE$. If $\angle EAB = 42°$, $\angle CBA = 19°$, find $\angle DCB$ in degrees.

下图中，$AB//DE$，$CD//AE$。若 $\angle EAB = 42°$，$\angle CBA = 19°$，求 $\angle DCB$ 的度数。



Answer: [61]

Solutions:



Draw CF to be parallel to AB.

$\angle FCB = \angle CBA = 19°$, $\angle DCF = \angle EAB = 42°$.

Hence, $\angle DCB = 61°$.

Question J-07 [5 points]

Ms Lee wants to recruit some students to move all the chairs in the hall to some classrooms. If she recruits 18 students, they will take 30 minutes to complete the job. If she wants the job to be completed in 20 minutes, how many students should Ms Lee recruit? It is assumed that every student moves the same number of chairs, and the time taken to move a chair is the same for each student.

李老师要叫一些学生来将礼堂里的椅子搬到教室去。如果她叫18位学生做这项工作，他们需要30分钟才能完成。如果李老师想在20分钟内完成这项任务，她需叫几位学生搬椅子? 这里是假设每位学生搬相同数目的椅子，每位学生搬动一张椅子所需的时间是一样的。

Answer: [27]

Solutions:

The amount of time taken is inversely proportional to the number of students doing the job. Hence, in order to complete the task in 20 minutes, Ms Lee need to recruit $\frac{18 \times 30}{20} = 27$ students.

Question J-08 [5 points]

Given that a regular n -gon has 299 diagonals, find the value of n.

已知一正的n边形有 299 条对角线，求n 的值。

Answer: [26]

Solutions:

$$\frac{n(n-3)}{2} = 299$$

$$n^2 - 3n - 23 \times 26 = 0$$

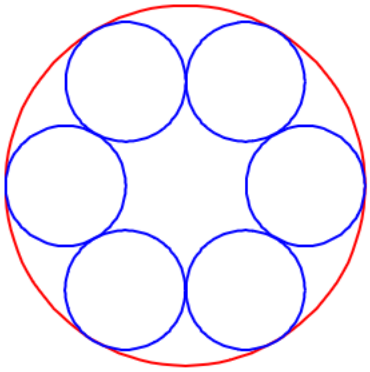
$$(n-26)(n+23) = 0$$

$$n = 26$$

Question J-09 [5 points]

In the figure below, the six small circles are inside the large circle and are tangent to it. Each of the small circles are also tangent to the two small circles beside it. Assume that the area of the large circle is S_1, and the sum of the areas of the six small circles is S_2. Find $60\dfrac{S_1}{S_2}$.

下图中，六个小圆在大圆里，且均与大圆相切。任一个小圆均与其相邻的两个小圆相切。若大圆的面积是 S_1，六个小圆的面积之和是 S_2，求 $60\dfrac{S_1}{S_2}$。



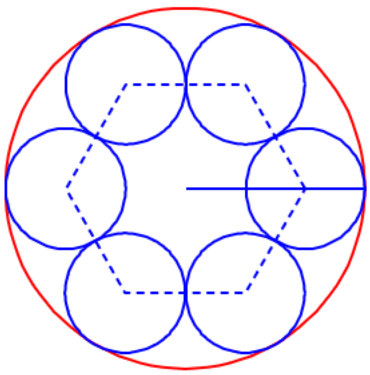
Answer: [90]

Solutions:

If the radius of a small circle is r, then the radius of the large circle is $3r$.

Hence,

$$60\frac{S_1}{S_2} = 60 \times \frac{\pi(3r)^2}{6 \times \pi r^2} = 90$$



Question J-10 [5 points]

Given that the five-digit number $\overline{2a5a6}$ is divisible by 36, find the digit a.

已知五位数 $\overline{2a5a6}$ 可以被 36 整除，求数字 a。

Answer: [7]

Solutions:

A number is divisible by 36 if and only if it is divisible by 4 and 9.

$\overline{2a5a6}$ is divisible by 4, so $\overline{a6}$ must be divisible by 4, and hence it can only be 1, 3, 5, 7, 9.

$\overline{2a5a6}$ is divisible by 9, so $2 + a + 5 + a + 6 = 13 + 2a$ is divisible by 9.

Hence, a must be 7.

Question J-11 [5 points]

Given that a is an integer. Find the value of $\dfrac{7943}{a^2-(a+13)(a-13)}$.

已知 a 是一整数。求 $\dfrac{7943}{a^2-(a+13)(a-13)}$ 的值。

Answer: [47]

Solutions:

$$\frac{7943}{a^2-(a+13)(a-13)}=\frac{7943}{13^2}=47$$

Question J-12 [5 points]

Among the integers from 100 to 1000, how many of them are multiples of 13?

由 100 到 1000 的整数中有多少个 13 的倍数?

Answer: [69]

Solutions:

$$\left\lfloor \frac{1000}{13} \right\rfloor = 76, \qquad \left\lfloor \frac{100}{13} \right\rfloor = 7.$$

Therefore, there are 69 integers from 100 to 1000 which are multiples of 13.

Question J-13 [5 points]

If x and y are two numbers such that the value of $143x - 77y$ is 451, find the value of $299x - 161y$.

若 x 及 y 这两个数使得 $143x - 77y$ 的值等于451，求 $299x - 161y$ 的值。

Answer: [943]

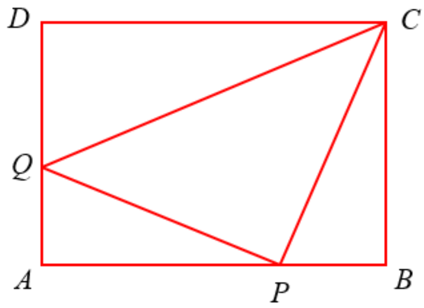
Solutions:

$$\begin{aligned} 143x - 77y &= 451 \\ 13x - 7y &= 41 \\ 299x - 161y &= 23(13x - 7y) = 943 \end{aligned}$$

Question J-14 [5 points]

In the figure shown below, $ABCD$ is a rectangle with area 780. Given that $AP : PB = 9 : 4$, and $AQ : QD = 2 : 3$, find the area of ΔCPQ.

下图中，$ABCD$ 是一个面积等于 780 的长方形。已知 $AP : PB = 9 : 4$，而 $AQ : QD = 2 : 3$，求 ΔCPQ 的面积。



Answer: [318]

Solutions:

Let $AB = x$, $AD = y$, then $xy = 780$, $AP = \frac{9}{13}x$, $PB = \frac{4}{13}$, $AQ = \frac{2}{5}y$, $QD = \frac{3}{5}y$. Hence, the area of ΔCPQ is

$$
\begin{aligned}
& xy - \frac{1}{2} \times \frac{9}{13}x \times \frac{2}{5}y - \frac{1}{2} \times \frac{4}{13}x \times y - \frac{1}{2} \times x \times \frac{3}{5}y \\
=& 780 \left(1 - \frac{18 + 20 + 39}{130}\right) \\
=& 318
\end{aligned}
$$

Question J-15 [5 points]

In Malaysia, some people write 7$^{\text{th}}$ of March as 07/03, using the dd/mm notation, but some people write it as 03/07, using the mm/dd notation. Hence, when we see a date written as 07/03, we don't know whether it means 7$^{\text{th}}$ of March or 3$^{\text{rd}}$ of July. However, there is no ambiguity when we see a date written as 28/02 or 02/28, since it is obvious that both mean 28$^{\text{th}}$ of February, as there are only twelve months in a year. In the year 2021, how many of the days would create confusion when it is not certain whether the date is written in dd/mm or mm/dd notations?

在马来西亚，有些人用 dd/mm 的格式将 3 月 7 日写成 07/03，也有些人用 mm/dd 的格式将它写成 03/07。因此，当我们看到一个日期写成 07/03 时，我们无法判断这是 3 月 7 号还是 7 月 3 号。可是当我们看到 28/02 或 02/28，我们就知道两者都是 2 月 28 号，因为一年只有 12 个月。在 2021 这一年，有多少天会因为不确定它是以 dd/mm 或 mm/dd 的格式来写而照成混淆?

Answer: [132]

Solutions:

The n^{th} day of the m^{th} month will give rise to confusion if $1 \leq m \leq 12$, $1 \leq n \leq 12$ but $m \neq n$. Hence, there are 11 days in each month that will give rise to confusion. Therefore, there are a total of $12 \times 11 = 132$ days in 2021 that would give rise to confusion.

Question J-16 [5 points]

Given that a, b, c are three positive integers. The least common multiple of a and b is $2^2 \times 3^4 \times 7^2$, the least common multiple of b and c is $2^3 \times 3^3 \times 5 \times 7$. Find the largest possible value of b.

已知a，b，c是三个正整数。a 与b 的最小公倍数是$2^2 \times 3^4 \times 7^2$，$b$与$c$的最小公倍数是$2^3 \times 3^3 \times 5 \times 7$，求$b$的最大可能值。

Answer: [756]

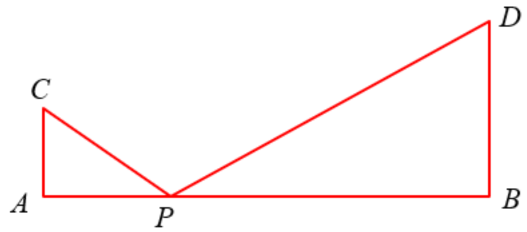
Solutions:

b must be divisible by $2^2 \times 3^4 \times 7^2$ and $2^3 \times 3^3 \times 5 \times 7$. Hence, the largest possible value of is $2^2 \times 3^3 \times 7 = 756$.

Question J-17 [5 points]

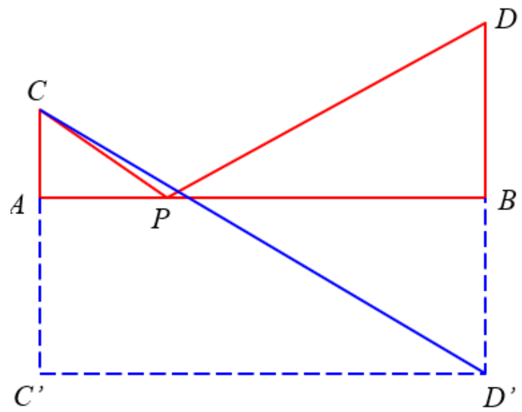
In the figure below, AC and BD are perpendicular to AB, $AC = 102$, $BD = 201$, $AB = 404$. Given that P is a point on the line segment AB, find the minimum possible value of $PC+PD$.

下图中，AC 与 BD 分别垂直于 AB，$AC = 102$，$BD = 201$，$AB = 404$。已知 P 是线段 AB 上的一点，求 $PC + PD$ 的最小可能值。



Answer: [505]

Solutions:



As shown in the figure above, extend CA and DB to C' and D' respectively so that $C'A = D'B = DB$. Then $PC + PD = PC + PD'$ is minimum when P is the intersection of CD' with AB. The minimum value is CD'.

$$CC' = 102 + 201 = 303$$

$$C'D' = AB = 404$$

$$CD' = \sqrt{303^2 + 404^2} = 505$$

Question J-18 [5 points]

There are 76 people in a conference, coming from two universities, university A and university B. The first person from university A shakes hands with 19 people from university B, the second person from university A shakes hands with 20 people from university B, the third person from university A shakes hands with 21 people from university B. This pattern follows up to the last person from university A, who shakes hands with all the people from university B. How many people are there from university B?

有76人参加一个会议，他们来自大学A及大学B。大学A的第一位代表和大学B的19位代表握手，大学A的第二位代表和大学B的20位代表握手，大学A的第三位代表和大学B的21位代表握手。按照这样的规律，大学A的最后一位代表和大学B的所有代表都握手。问大学B有多少位代表？

Answer: [47]

Solutions:

If n people are from university B, then $76 - n$ people are from university A. According to the stipulated pattern, the number of people shaking hands with the last person, who is the $(76 - n)^{\text{th}}$ person, from university A is $76 - n + 18$, and this is equal to n.

$$76 - n + 18 = n$$

$$n = 47$$

Question J-19 [5 points]

There were six teams of students in a sports event. If a total of 183 gold medals were won by these six teams, and the number of gold medals team A members won was the highest among the six teams, more than any of the other five teams, what is the minimum number of gold medals team A members had won?

有六队学生参加运动会，一共赢了 183 面金牌。如果 A 队是六队中赢得最多金牌的，比其他五队的任何一队多，则 A 队至少赢得多少面金牌?

Answer: [32]

Solutions:

$$\frac{183}{6} = 30 + \frac{3}{6}$$

Hence, team A must win at least 31 gold medals.

If team A won 31 gold medals, each of the other five teams won at most 30 gold medals, then the total number of gold medals would not exceed 181.

Hence, team A had to win more than 31 gold medals.

It can happen that team A won 32 gold medals, one of the other five teams won 31 gold medals, and the remaining four teams won 30 gold medals each.

Hence, the minimum number of gold medals team A had won is 32.

Question J-20 [5 points]

Liping wants to travel from town A to town B by bicycle to meet her friend Xiaomei. Based on her previous experience, Liping knows if she bikes at a uniform speed of 10 km/h, she will reach town B at 6 pm, an hour later than the appointment time. If she bikes at a uniform speed of 15 km/h, she will reach town B at 4 pm, an hour earlier than the appointment time. What is the uniform speed in km/h that Liping should travel so that she can reach town B just on time?

丽萍要从市镇A骑脚踏车到市镇B去找她的朋友晓梅。根据以往的经验，丽萍知道如果她以每小时10 km 的均匀速度骑脚踏车，她将在下午6点到达市镇B，比约定的时间晚一个小时；如果她以每小时15 km 的均匀速度骑脚踏车，她将在下午4点到达市镇B，比约定的时间早一个小时。如果丽萍想要在约定的时间准时到达市镇B，她应该以每小时多少 km 的均匀速度骑脚踏车？

Answer: [12]

Solutions:

Assume that the distance between town A and town B is s km, and the time from departure to 5 pm, the appointment time, is h hours. Then

$$\begin{aligned} \frac{s}{10} &= h + 1 \\ \frac{s}{15} &= h - 1 \\ 2h &= \frac{s}{6} \\ h &= \frac{s}{12} \end{aligned}$$

Hence, Liping should travel at a uniform speed of 12 km/h.

Question J-21 [6 points]

How many ways are there to divide 6 people into two groups so that each group has 3 people?

有多少种方法可以将6个人分成两组，每组3个人?

Answer: [10]

Solutions:

First assume that the two groups are different. There are 6 ways to choose the first person in the first group, 5 ways to choose the second person, 4 ways to choose the third person. There are $6 \times 5 \times 4 = 6 \times 20$ ways to choose three persons in order. Since the order of choosing these three people is not important, and there are $3 \times 2 = 6$ ways to order 3 things, there are only 20 ways to choose the first group. Now the two groups are indistinguishable. Hence, there are only 10 ways to divide the 6 people into two groups, 3 people each.

Question J-22 [6 points]

If x is a real number such that $x^2 - 7x + 2 = 0$, find the value of $x^3 + \frac{8}{x^3}$.

若 x 是实数且 $x^2 - 7x + 2 = 0$，求 $x^3 + \frac{8}{x^3}$ 的值。

Answer: [301]

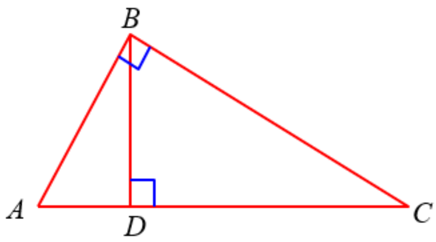
Solutions:

$$
\begin{aligned}
x + \frac{2}{x} &= 7 \\
x^2 + 4 + \frac{4}{x^2} &= 49 \\
x^2 + \frac{4}{x^2} &= 45 \\
\left(x + \frac{2}{x}\right)\left(x^2 + \frac{4}{x^2}\right) &= 7 \times 45 \\
x^3 + \frac{8}{x^3} + 2\left(x + \frac{2}{x}\right) &= 315 \\
x^3 + \frac{8}{x^3} &= 301
\end{aligned}
$$

Question J-23 [6 points]

In the figure below, $\angle ABC = \angle ADB = 90°$. If $AD = 9$, $BC = 20$, find BD.

下图中，$\angle ABC = \angle ADB = 90°$。若 $AD = 9$，$BC = 20$，求 BD。



Answer: [12]

Solutions:

Let $BD = x$. Then $CD = \sqrt{400 - x^2}$.

$$\frac{AD}{BD} = \frac{BD}{CD}$$
$$\frac{9}{x} = \frac{x}{\sqrt{400 - x^2}}$$
$$x^2 = 9\sqrt{400 - x^2}$$
$$x^4 = 81(400 - x^2)$$
$$x^4 + 81x^2 - 81 \times 400 = 0$$
$$(x^2 - 9 \times 16)(x^2 + 9 \times 25) = 0$$
$$x^2 = 9 \times 16$$
$$x = 3 \times 4 = 12$$

Question J-24 [6 points]

If m and n are positive integers such that $11m + 101n = 986$, find the smallest value of $m + n$.

若 m 及 n 是正整数且 $11m + 101n = 986$，求 $m + n$ 的最小值。

Answer: [16]

Solutions:

$$11m + 101n = 986$$
$$11(m + 9n) + 2n = 990 - 4$$
$$2(n + 2) = 11(90 - m - 9n)$$

This implies that $n + 2$ is divisible by 11.

Writing $n + 2 = 11k$, with k an integer, we find that

$$90 - m - 9n = 2k$$

$$m = 90 - 9(11k - 2) - 2k = 108 - 101k$$

Now in order that $n = 11k - 2$ is positive, we require that $k \geq 1$.

In order that $m = 108 - 101k$ is positive, we require that $k \leq 1$.

Hence, $k = 1$, $m = 7$, $n = 9$, $m + n = 16$.

Question J-25 [6 points]

Given the four-digit number $\overline{2ab3}$, we reverse the order of the two digits in the middle and form the four-digit number $\overline{2ba3}$. Find the maximum possible value of $\overline{2ab3} - \overline{2ba3}$, the difference of these two numbers.

将四位数$\overline{2ab3}$ 的中间两位数字调转过来就得到四位数 $\overline{2ba3}$。求这两个四位数的差，$\overline{2ab3} - \overline{2ba3}$，的最大可能值。

Answer: [810]

Solutions:

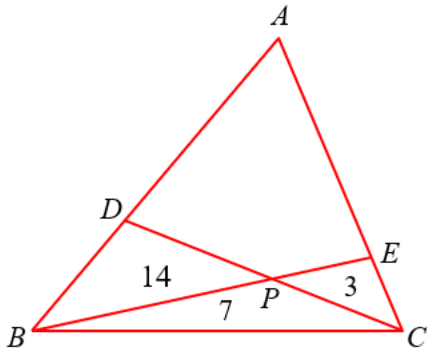
$$\begin{aligned}\overline{2ab3} - \overline{2ba3} &= \left(2 \times 10^3 + a \times 10^2 + b \times 10 + 3\right) - \left(2 \times 10^3 + b \times 10^2 + a \times 10 + 3\right) \\ &= 100(a-b) - 10(a-b) \\ &= 90(a-b)\end{aligned}$$

Since a and b are both integers that can take values from 0 to 9, the maximum value of $a - b$ is 9, which appears when $a = 9$, $b = 0$. Hence, the maximum value of $a - b$ is 810.

Question J-26 [8 points]

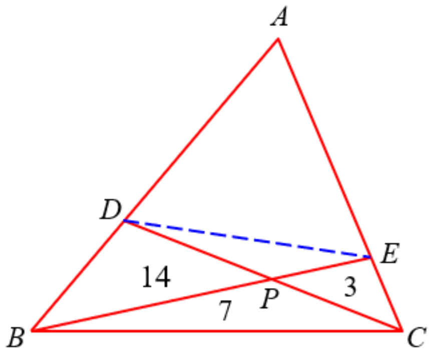
In the figure below, D and E are points on AB and AC respectively. The lines CD and BE intersect at the point P. If the areas of the triangles PBC, PCE and PBD are 7, 3 and 14 respectively, find the area of the quadrilateral $ADPE$.

下图中，D及E分别是AB及AC边上的点，直线CD及BE相交于点P。若三角形PBC，PCE 及PBD的面积分别为7，3及14，求四边形$ADPE$的面积。



Answer: [186]

Solutions:



$$\frac{S_{\Delta PDE}}{S_{\Delta PCE}} = \frac{PD}{PC} = \frac{S_{\Delta PBD}}{S_{\Delta PBC}} = 2$$

$$S_{\Delta PDE} = 6$$

$$\frac{S_{\Delta ADE}}{S_{\Delta BDE}} = \frac{AD}{BD} = \frac{S_{\Delta ACD}}{S_{\Delta BCD}}$$

$$\frac{S_{\Delta ADE}}{20} = \frac{S_{\Delta ADE} + 9}{21}$$

$$21S_{\Delta ADE} = 20S_{\Delta ADE} + 180$$

$$S_{\Delta ADE} = 180$$

$$S_{ADPE} = 186$$

Question J-27 [8 points]

Given that n is a positive integer and the greatest common divisor of $2n + 25$ and $3n + 2$ is greater than 1, find the smallest possible value of n.

已知 n 是一正整数，且 $2n + 25$ 与 $3n + 2$ 的最大公因数大于 1，求 n 的最小可能值。

Answer: [23]

Solutions:

$$3(2n + 25) - 2(3n + 2) = 71$$

If d is the greatest common divisor of $2n + 25$ and $3n + 2$, d divides these two numbers, hence it divides 71. Since 71 is a prime number, and $d > 1$, we find that $d = 71$. The integer solutions to $3m - 2k = 71$ must be of the form $m = 71 + 2h$, $n = 71 + 3h$ for some integer h. Since $2n + 25$ and $3n + 2$ are divisible by 71, we find that

$$2n + 25 = 71 + 2h$$
$$3n + 2 = 71 + 3h$$

and h must be divisible by 71.

Hence, the smallest positive n appears when $h = 0$, so that $n = 23$.

Question J-28 [8 points]

How many positive integers n are there such that $\left\lfloor \sqrt{2n} + \frac{7}{2} \right\rfloor = 789$?
Here $\lfloor x \rfloor$ is the largest integer less than or equal to x.

有多少个正整数 n 使得 $\left\lfloor \sqrt{2n} + \frac{7}{2} \right\rfloor = 789$?
$\lfloor x \rfloor$ 是小于或等于 x 的最大整数。

Answer: [786]

Solutions:

$$\left\lfloor \sqrt{2n} + \frac{7}{2} \right\rfloor = 789$$
$$789 \leq \sqrt{2n} + \frac{7}{2} < 790$$
$$786 - \frac{1}{2} \leq \sqrt{2n} < 786 + \frac{1}{2}$$
$$\left(786 - \frac{1}{2}\right)^2 \leq 2n < \left(786 + \frac{1}{2}\right)^2$$
$$786^2 - 786 + \frac{1}{4} \leq 2n < 786^2 + 786 + \frac{1}{4}$$
$$\frac{786^2 - 786}{2} + 1 \leq n \leq \frac{786^2 + 786}{2}$$

There are

$$\frac{786^2 + 786}{2} - \frac{786^2 - 786}{2} = 786$$

such integers n.

Question J-29 [8 points]

Given that n is a positive integer and $2^{714}+2^{717}+2^n$ is a perfect square, find the value of n.

已知n是一正整数且 $2^{714}+2^{717}+2^n$ 是一平方数，求n的值。

Answer: [718]

Solutions:

If $1 \le n < 714$,

$$2^{714}+2^{717}+2^n = 2^n\left(1+9\times 2^{714-n}\right)$$

$1+2^{714-n}$ is an odd number.

Hence, for $2^n\left(1+9\times 2^{714-n}\right)$ to be a square number, n must be even, and $1+9\times 2^{714-n}$ is the square of an odd number.

But $9\times 2^{714-n}$ is a square number as well.

It is impossible to have two consecutive positive square numbers.

If $n = 714$,

$$2^{714}+2^{717}+2^n = 10\times 2^{714}$$

This cannot be a square number.

If $n > 714$,

$$2^{714}+2^{717}+2^n = 2^{714}\left(9+2^{n-714}\right)$$

If this is a square number, since 2^{714} is a square number, $9+2^{n-714}$ is also a square number.

$$9+2^{n-714} = m^2$$

$$m^2-9 = 2^{n-714}$$

$$(m-3)(m+3) = 2^{n-714}$$

Therefore, there exists nonnegative integers k and l such that $k<l$ and

$$m-3 = 2^k$$

$$m+3 = 2^l$$

$$2^k(2^{l-k}-1) = 6$$

Therefore, $k=1$, $l=3$,

$$2^{n-714} = 16$$

Hence, $n-714=4$ and therefore $n=718$.

Question J-30 [8 points]

There are six boys and six girls coming from three families, with each family having two boys and two girls. They want to form 6 pairs of mixed doubles to take part in a badminton tournament, each pair having one boy and one girl. If one of the rules of the tournament is that the players in a mixed pair cannot come from the same family, how many ways can the six mixed pairs be formed?

有六男六女来自三个家庭，每个家庭二男二女。他们要组成六对混双，每对一男一女，去参加羽毛球比赛。如果比赛规则说明每对混双的队员不能来自同一个家庭，则有多少种方法可以组成这六对混双?

Answer: [80]

Solutions:

Assume that the three families are family 1, family 2 and family 3.

There are two cases.

The first case is the partners of each boy from a family are the girls from the same family.

Namely, it can be that

- The boys from family 1 are paired with the girls from family 2, then the boys from family 2 are paired with the girls from family 3, and the boys from family 3 are paired with the girls from family 1.

  or

- The boys from family 1 are paired with the girls from family 3, then the boys from family 2 are paired with the girls from family 1, and the boys from family 3 are paired with the girls from family 2.

Since there are two ways to pair the boys from family i with the girls from family j, we find that there are a total of $2 \times 2 \times 2 \times 2 = 16$ ways in this case.

The second case is the partners of each boy from a family come from different families.

Hence, to pair with the two boys from family 1, we need to choose a girl from family 2, which has 2 ways, and a girl from family 3, which has 2 ways, and there are 2 ways of pairing. Then to pair with the two boys from family 2, there are two ways to choose a girl from family 1, but there is only 1 way left to pick the girl from family 3, and there are 2 ways of pairing. Finally, for the two boys from family 3, they need to be paired with the remaining two girls, one from family 1 and one from family 2, there are 2 ways.

Hence, there are

$$(2 \times 2 \times 2 \times 2) \times (2 \times 2) \times 2 = 64$$

ways of pairing in this case.

This gives a total of $16 + 64 = 80$ ways of forming the six mixed double pairs.