厦门大学马来西亚分校
陈景润杯中学数学竞赛

CHEN JINGRUN'S CUP SECONDARY SCHOOL MATHEMATICS COMPETITION

JUNIOR CATEGORY

** 初阶组 **

Date: 1st April 2021 Time: 10:00 a.m. to 12:30 p.m.

日期：2021 年 4 月 1 日 时间：上午 10 时至下午 12 时 30 分

考生须知

Information and Instruction

这是**初阶组**的题目的线下版本。
This is the off-line version of the question paper for the **JUNIOR** category.

负责老师会发给你一个 Microsoft Form 的链接，你必须上线在 Form 上填写答案提交。请务必在正确的 Form 上提交答案。
You need to go online to enter the answers to a Microsoft Form whose link is shared to you by your Teacher in Charge. Make sure you submit the correct form.

Microsoft Form 将在下午 12 点 30 分关闭，请在 12 点 25 分之前就按下“提交”键，以免因为网络问题而无法提交。
The Microsoft Form would be closed at 12:30pm. Please press the "Submit" button before 12:25pm, to avoid failure of submission due to network problems.

请仔细阅读下一页的资讯与指示，再继续作答。
Read through the information and instructions given in next page carefully. Then proceed to answer the questions.

在 Microsoft Form 的最后一页，你必须填写个人资料，再提交表格。请务必填写正确的**考生编号**。考生编号不是学号，也不是身份证号码。它是主办方提供的编号。如果你不清楚自己的考生编号，请询问学校的负责老师。
You are required to enter your personal information in the last page of the Microsoft Form, before you submit the form. Please enter your **candidate ID** correctly. The candidate ID is not your student ID, and is also not your IC number. It is a unique code given by the organizer. If you do not know your candidate ID, please ask the teacher in charge of your school.

一共有 30 题，**每题的答案都是一个整数。**每题在一个页面。请在 Microsoft Form 相应的空格内填入答案，**答案必须是一个数字，不用写单位，不要填入数字以外的字元。**
There are altogether 30 questions. **The answer to each question is an integer.** Each question is on a single page. Please enter your answer in the corresponding empty box in the Microsoft Form. **The answer must be a number. Do not write the units. Do not enter characters other than numbers.**

第 1 至 第 20 题，每题 5 分	Question 1 to Question 20, each question carries 5 points
第 21 至 第 25 题，每题 6 分	Question 21 to Question 25, each question carries 6 points
第 26 至 第 30 题，每题 8 分	Question 26 to Question 30, each question carries 8 points
总分是 170 分	Total is 170 points.

学术诚信

Academic Honesty

参赛者必须独立参与比赛。不可得到其他人的协助，在比赛期间不能以任何方式和任何人互通信息。如果违反这些规则，参赛资格将被取消，而且也会受到学校的严厉纪律处分。

Participants should work independently in this competition. You should not get help from others, and should not communicate with anyone by any means during the competition. If you are found to violate these rules, you would be disqualified and face severe disciplinary action from your school.

关于线上提交答案的温馨提示

Kind Reminder for Submitting Answers to Microsoft Form online

1. "Submit"键在最后一页，在比赛时间结束前，请记得按下该键。
   The "Submit" button is at the last page. Before the end of the exam time, please remember to press the "Submit" button.

2. 有打 " * " 的项目一定要作答，否则无法提交。
   It is compulsory to response to all items with " * ", or else you would not be able to submit.

3. 未按“Submit" 键之前，一旦刷新或关掉浏览器，所有填入的资料将消失。
   If you refresh or close your browser before you press the "Submit" button, all the information that you have filled in will be lost.

4. 请在草稿纸上记录各题的答案，一旦不小心刷新或关掉浏览器，请重新登入 Microsoft Form 再填写一次。
   Please record your answers to each question on a piece of draft paper. If you accidentally refresh or close your browser, login the Microsoft form again to fill in all the answers.

5. 如果重复提交答案，必须提供合理的解释。
   If the form is submitted more than once, you need to give a good reason.

Question J-01 [5 points]

Today is Thursday. What is the day 2021 days later?

The code of the days are tabulated below. Enter the code as your answer.

今天是星期四。2021 天之后是星期几?

下表为各天的代码。请填入代码作为答案。

Day	天	Code /代码
Monday	星期一	1
Tuesday	星期二	2
Wednesday	星期三	3
Thursday	星期四	4
Friday	星期五	5
Saturday	星期六	6
Sunday	星期天	7

Question J-02 [5 points]

The average height of the two persons P and Q is 152 cm, while the average height of the five persons P, Q, R, S and T is 155 cm. What is the average height of the three persons R, S and T in cm?

P，Q 两人的平均身高是 152 cm，P，Q，R，S，T 五人的平均身高是 155 cm。R，S，T 三人的平均身高是多少 cm?

Question J-03 [5 points]

Xiaoming, Xiaolan, Wenying and Fenfen went to a stationery shop together. Xiaoming spent RM 3.20 to buy one pencil and one pen. Xiaolan spent RM 5.50 to buy one pencil and one exercise book. Wenying spent RM 4.30 to buy one pen and one ruler. If Fenfen bought 10 exercise books and 10 rulers, how much did she spend in RM?

小明，小兰，文英，芬芬四人一起去文具店。小明买一支铅笔和一支原子笔花了 RM 3.20，小兰买一支铅笔和一本作业簿花了 RM 5.50，文英买一支原子笔与一把尺花了 RM 4.30。若芬芬买了 10 本作业簿和 10 把尺，她花了多少令吉？

Question J-04 [5 points]

How many positive factors does 120 have?

120 有几个正的因数?

Question J-05 [5 points]

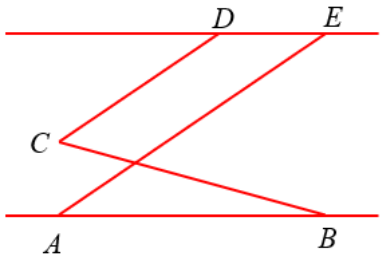
How many digits are there in the number $8^{10} \times 5^{26}$?

$8^{10} \times 5^{26}$ 这个数有几位数字?

Question J-06 [5 points]

In the figure below, $AB//DE$, $CD//AE$. If $\angle EAB = 42°$, $\angle CBA = 19°$, find $\angle DCB$ in degrees.

下图中，$AB//DE$，$CD//AE$。若 $\angle EAB = 42°$，$\angle CBA = 19°$，求 $\angle DCB$ 的度数。



Question J-07 [5 points]

Ms Lee wants to recruit some students to move all the chairs in the hall to some classrooms. If she recruits 18 students, they will take 30 minutes to complete the job. If she wants the job to be completed in 20 minutes, how many students should Ms Lee recruit? It is assumed that every student moves the same number of chairs, and the time taken to move a chair is the same for each student.

李老师要叫一些学生来将礼堂里的椅子搬到教室去。如果她叫18位学生做这项工作，他们需要30分钟才能完成。如果李老师想在20分钟内完成这项任务，她需叫几位学生搬椅子？这里是假设每位学生搬相同数目的椅子，每位学生搬动一张椅子所需的时间是一样的。

Question J-08 [5 points]

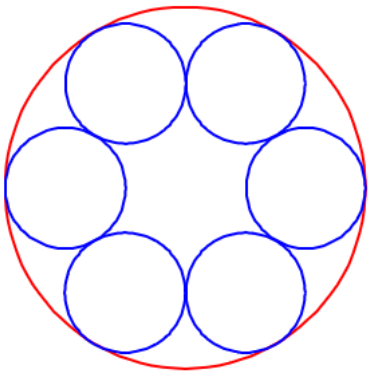
Given that a regular n -gon has 299 diagonals, find the value of n.

已知一正的 n 边形有 299 条对角线，求 n 的值。

Question J-09 [5 points]

In the figure below, the six small circles are inside the large circle and are tangent to it. Each of the small circles are also tangent to the two small circles beside it. Assume that the area of the large circle is S_1, and the sum of the areas of the six small circles is S_2. Find $60\dfrac{S_1}{S_2}$.

下图中，六个小圆在大圆里，且均与大圆相切。任一个小圆均与其相邻的两个小圆相切。若大圆的面积是 S_1，六个小圆的面积之和是 S_2，求 $60\dfrac{S_1}{S_2}$。



Question J-10 [5 points]

Given that the five-digit number $\overline{2a5a6}$ is divisible by 36, find the digit a.

已知五位数 $\overline{2a5a6}$ 可以被 36 整除，求数字 a。

Question J-11 [5 points]

Given that a is an integer. Find the value of $\dfrac{7943}{a^2-(a+13)(a-13)}$.

已知 a 是一整数。求 $\dfrac{7943}{a^2-(a+13)(a-13)}$ 的值。

Question J-12 [5 points]

Among the integers from 100 to 1000, how many of them are multiples of 13?

由 100 到 1000 的整数中有多少个 13 的倍数?

Question J-13 [5 points]

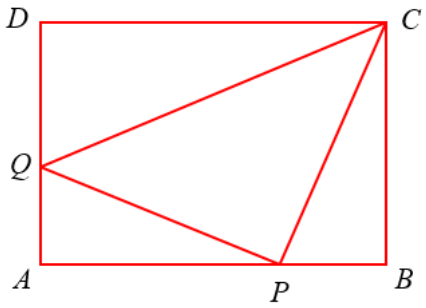
If x and y are two numbers such that the value of $143x - 77y$ is 451, find the value of $299x - 161y$.

若 x 及 y 这两个数使得 $143x - 77y$ 的值等于451，求 $299x - 161y$ 的值。

Question J-14 [5 points]

In the figure shown below, $ABCD$ is a rectangle with area 780. Given that $AP : PB = 9 : 4$, and $AQ : QD = 2 : 3$, find the area of ΔCPQ.

下图中，$ABCD$ 是一个面积等于 780 的长方形。已知 $AP : PB = 9 : 4$，而 $AQ : QD = 2 : 3$，求 ΔCPQ 的面积。



Question J-15 [5 points]

In Malaysia, some people write 7th of March as 07/03, using the dd/mm notation, but some people write it as 03/07, using the mm/dd notation. Hence, when we see a date written as 07/03, we don't know whether it means 7th of March or 3rd of July. However, there is no ambiguity when we see a date written as 28/02 or 02/28, since it is obvious that both mean 28th of February, as there are only twelve months in a year. In the year 2021, how many of the days would create confusion when it is not certain whether the date is written in dd/mm or mm/dd notations?

在马来西亚，有些人用 dd/mm 的格式将 3 月 7 日写成 07/03，也有些人用 mm/dd 的格式将它写成 03/07。因此，当我们看到一个日期写成 07/03 时，我们无法判断这是 3 月 7 号还是 7 月 3 号。可是当我们看到 28/02 或 02/28，我们就知道两者都是 2 月 28 号，因为一年只有 12 个月。在 2021 这一年，有多少天会因为不确定它是以 dd/mm 或 mm/dd 的格式来写而照成混淆?

Question J-16 [5 points]

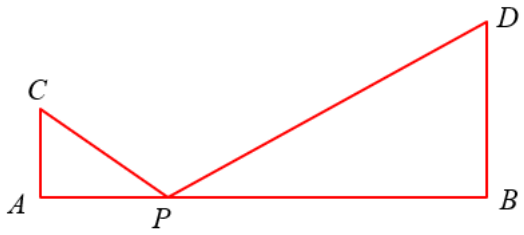
Given that a, b, c are three positive integers. The least common multiple of a and b is $2^2 \times 3^4 \times 7^2$, the least common multiple of b and c is $2^3 \times 3^3 \times 5 \times 7$. Find the largest possible value of b.

已知a，b，c是三个正整数。a 与b 的最小公倍数是$2^2 \times 3^4 \times 7^2$，$b$与$c$的最小公倍数是$2^3 \times 3^3 \times 5 \times 7$，求$b$的最大可能值。

Question J-17 [5 points]

In the figure below, AC and BD are perpendicular to AB, $AC = 102$, $BD = 201$, $AB = 404$. Given that P is a point on the line segment AB, find the minimum possible value of $PC + PD$.

下图中，AC 与 BD 分别垂直于 AB，$AC = 102$，$BD = 201$，$AB = 404$。已知 P 是线段 AB 上的一点，求 $PC + PD$ 的最小可能值。



Question J-18 [5 points]

There are 76 people in a conference, coming from two universities, university A and university B. The first person from university A shakes hands with 19 people from university B, the second person from university A shakes hands with 20 people from university B, the third person from university A shakes hands with 21 people from university B. This pattern follows up to the last person from university A, who shakes hands with all the people from university B. How many people are there from university B?

有76人参加一个会议，他们来自大学A及大学B。大学A的第一位代表和大学B的19位代表握手，大学A的第二位代表和大学B的20位代表握手，大学A的第三位代表和大学B的21位代表握手。按照这样的规律，大学A的最后一位代表和大学B的所有代表都握手。问大学B有多少位代表？

Question J-19 [5 points]

There were six teams of students in a sports event. If a total of 183 gold medals were won by these six teams, and the number of gold medals team A members won was the highest among the six teams, more than any of the other five teams, what is the minimum number of gold medals team A members had won?

有六队学生参加运动会，一共赢了 183 面金牌。如果 A 队是六队中赢得最多金牌的，比其他五队的任何一队多，则 A 队至少赢得多少面金牌?

Question J-20 [5 points]

Liping wants to travel from town A to town B by bicycle to meet her friend Xiaomei. Based on her previous experience, Liping knows if she bikes at a uniform speed of 10 km/h, she will reach town B at 6 pm, an hour later than the appointment time. If she bikes at a uniform speed of 15 km/h, she will reach town B at 4 pm, an hour earlier than the appointment time. What is the uniform speed in km/h that Liping should travel so that she can reach town B just on time?

丽萍要从市镇A骑脚踏车到市镇B去找她的朋友晓梅。根据以往的经验，丽萍知道如果她以每小时10 km 的均匀速度骑脚踏车，她将在下午6点到达市镇B，比约定的时间晚一个小时；如果她以每小时15 km 的均匀速度骑脚踏车，她将在下午4点到达市镇B，比约定的时间早一个小时。如果丽萍想要在约定的时间准时到达市镇B，她应该以每小时多少 km 的均匀速度骑脚踏车？

Question J-21 [6 points]

How many ways are there to divide 6 people into two groups so that each group has 3 people?

有多少种方法可以将6个人分成两组，每组3个人?

Question J-22 [6 points]

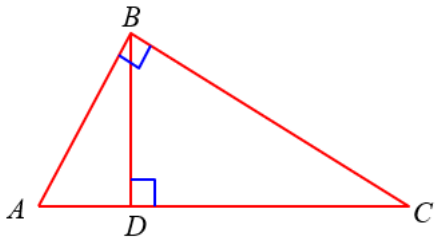
If x is a real number such that $x^2 - 7x + 2 = 0$, find the value of $x^3 + \dfrac{8}{x^3}$.

若 x 是实数且 $x^2 - 7x + 2 = 0$，求 $x^3 + \dfrac{8}{x^3}$ 的值。

Question J-23 [6 points]

In the figure below, $\angle ABC = \angle ADB = 90°$. If $AD = 9$, $BC = 20$, find BD.

下图中，$\angle ABC = \angle ADB = 90°$。若 $AD = 9$，$BC = 20$，求 BD。



Question J-24 [6 points]

If m and n are positive integers such that $11m + 101n = 986$, find the smallest value of $m + n$.

若 m 及 n 是正整数且 $11m + 101n = 986$，求 $m + n$ 的最小值。

Question J-25 [6 points]

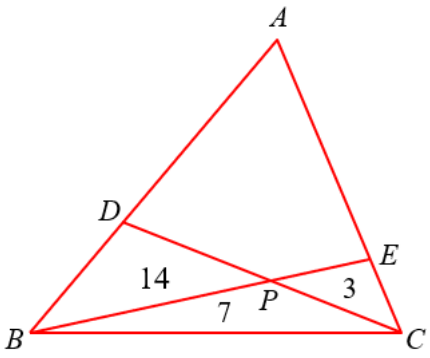
Given the four-digit number $\overline{2ab3}$, we reverse the order of the two digits in the middle and form the four-digit number $\overline{2ba3}$. Find the maximum possible value of $\overline{2ab3} - \overline{2ba3}$, the difference of these two numbers.

将四位数 $\overline{2ab3}$ 的中间两位数字调转过来就得到四位数 $\overline{2ba3}$。求这两个四位数的差，$\overline{2ab3} - \overline{2ba3}$，的最大可能值。

Question J-26 [8 points]

In the figure below, D and E are points on AB and AC respectively. The lines CD and BE intersect at the point P. If the areas of the triangles PBC, PCE and PBD are 7, 3 and 14 respectively, find the area of the quadrilateral $ADPE$.

下图中，D及E分别是AB及AC边上的点，直线CD及BE相交于点P。若三角形PBC，PCE及PBD的面积分别为7，3及14，求四边形$ADPE$的面积。



Question J-27 [8 points]

Given that n is a positive integer and the greatest common divisor of $2n + 25$ and $3n + 2$ is greater than 1, find the smallest possible value of n.

已知 n 是一正整数，且 $2n + 25$ 与 $3n + 2$ 的最大公因数大于 1，求 n 的最小可能值。

Question J-28 [8 points]

How many positive integers n are there such that $\left\lfloor \sqrt{2n} + \frac{7}{2} \right\rfloor = 789$?

Here $\lfloor x \rfloor$ is the largest integer less than or equal to x.

有多少个正整数 n 使得 $\left\lfloor \sqrt{2n} + \frac{7}{2} \right\rfloor = 789$?

$\lfloor x \rfloor$ 是小于或等于 x 的最大整数。

Question J-29 [8 points]

Given that n is a positive integer and $2^{714} + 2^{717} + 2^n$ is a perfect square, find the value of n.

已知 n 是一正整数且 $2^{714} + 2^{717} + 2^n$ 是一平方数，求 n 的值。

Question J-30 [8 points]

There are six boys and six girls coming from three families, with each family having two boys and two girls. They want to form 6 pairs of mixed doubles to take part in a badminton tournament, each pair having one boy and one girl. If one of the rules of the tournament is that the players in a mixed pair cannot come from the same family, how many ways can the six mixed pairs be formed?

有六男六女来自三个家庭，每个家庭二男二女。他们要组成六对混双，每对一男一女，去参加羽毛球比赛。如果比赛规则说明每对混双的队员不能来自同一个家庭，则有多少种方法可以组成这六对混双?