厦门大学马来西亚分校

陈景润杯中学数学比赛

2018 年第 2 届陈景润杯中学数学比赛

~ 中阶组 ~

日期：2018 年 4 月 21 日

Date: 21st April 2018

时间：上午 10 时至中午 12 时

Time: 10:00 a.m. to 12:00 p.m.

考生须知

Instructions and Information

1. 本试卷共有 30 题。
   This paper contains 30 questions.
   - 第 1 题至第 10 题，选择题，每题 4 分。
     Question 1 to Question 10, multiple choice questions, each question carries 4 marks.
   - 第 11 题至第 30 题，问答题，每题的答案是一个介于 0 至 1000 之间的整数。
     Question 11 to Question 30, short questions. For each question, the answer is an integer between 0 and 1000.
     - 第 11 题至第 20 题每题 5 分。
       Question 11 to Question 20, each question carries 5 marks.
     - 第 21 题至第 25 题每题 6 分。
       Question 21 to Question 25, each question carries 6 marks.
     - 第 26 题至第 30 题每题 8 分。
       Question 26 to Question 30, each question carries 8 marks.
2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题，只需填写 A，B，C，D 或 E 作为答案。每题只能填入一个答案，否则以答错论。
   Please use 2B lead pencils to write your answer to each question in the appropriate space provided on the answer sheet. For a multiple choice question, you only need to write A, B, C, D or E as answer. Only one answer is accepted for each question, otherwise no credits would be given.
3. 所有的图形并没有按照比例作图，只作为辅助之用。
   All the diagrams are not drawn to scales. They are intended only as aids.
4. 不许使用计算器，数学工具，手机或其他计算器。
   No calculators, maths stencils, mobile phones or other calculating aids are permitted.
5. 在答案纸上清楚写上姓名，考生编号，学校名称及在学年级。
   Write your name, candidate number, school name and year of study clearly on the answer sheet.
6. 在监考老师宣布比赛开始之后，才可以翻开此考卷开始作答。
   You can only open this question booklet to start answering questions after the invigilator announces the beginning of the competition.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中，$\lfloor x \rfloor$ 表示小于或等于 x 的最大整数。

例如：$\lfloor 2 \rfloor = 2$，$\lfloor -2 \rfloor = -2$，$\lfloor 2.6 \rfloor = 2$，$\lfloor -2.6 \rfloor = -3$。

In this paper, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题，选择题，每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 已知 $a = \sqrt{50}$，求 $\sqrt{2}$。

   Given that $a = \sqrt{50}$, find $\sqrt{2}$.

   A. $\dfrac{10}{a}$ B. $\dfrac{100}{a}$ C. $\dfrac{10}{\sqrt{a}}$ D. $\dfrac{100}{a^2}$ E. $\dfrac{10}{a^2}$

2. V，W，X，Y，Z 五个小孩子分别有 v，w，x，y，z 粒糖果。w 比 v 多 30%，x 比 v 少 30%，y 比 w 少 30%，z 比 x 多 30%。下列何者是**正确**的？

   Five kids V, W, X, Y, Z have respectively v, w, x, y, z sweets. w is 30% more than v, x is 30% less than v, y is 30% less than w, z is 30% more than x. Which of the following is **TRUE**?

   A. Y 的糖果比 V 多，Z 的糖果比 V 少。
      Y has more sweets than V, Z has less sweets than V.
   B. Z 的糖果比 V 多，Y 的糖果比 V 少。
      Z has more sweets than V, Y has less sweets than V.
   C. Y 与 Z 的糖果一样多，但比 V 的少。
      Y and Z have the same number of sweets, but they have less sweets than V.
   D. Y 与 Z 的糖果一样多，且比 V 的多。
      Y and Z have the same number of sweets, and they have more sweets than V.
   E. Y，Z 与 V 有相同的糖果数。
      Y, Z and V have the same number of sweets.

3. 若 $4^x = 2^{2018} + 2^{2018} + 2^{2018} + 2^{2018}$，求 x。
If $4^x = 2^{2018} + 2^{2018} + 2^{2018} + 2^{2018}$, find x.

A. 1009 B. 1010 C. 1011 D. 2019 E. 2020

4. A，B 两个容器都装有醋与水的混合液体，A，B 两个容器中液体的体积之比是 $4:5$。容器 A 中醋与水的比例是 $2:3$，容器 B 中醋与水的比例是 $3:2$。将容器 A 与 B 中的液体都倒入另一空容器 C 中，求此时容器 C 中醋与水的比例。
Two containers A and B both contain mixtures of vinegar and water. The volumes of the liquid in the containers A and B are in the ratio $4:5$. The ratio of vinegar to water in container A is $2:3$, while the ratio of vinegar to water in container B is $3:2$. The liquid in containers A and B are poured into an empty container C. Find the ratio of vinegar to water in container C.

A. $1:1$ B. $4:5$ C. $2:3$ D. $12:11$ E. $23:22$

5. 一项考试结束后，张老师可以请爱慧，碧蓝，及岑月三位助理帮忙改卷子。若只有爱慧及碧蓝改，需要 20 小时才能完成。若只有爱慧及岑月改，需要 30 小时才能完成。若只有碧蓝及岑月改，需要 15 小时才能完成。问爱慧一个人改需要多少小时才能完成?
After an examination, Ms Zhang can ask her three assistants Aihui, Bilan and Cenyue to do the grading. If only Aihui and Bilan are asked to grade, it takes 20 hours to complete. If only Aihui and Cenyue are asked to grade, it takes 30 hours to complete. If only Bilan and Cenyue are asked to grade, it takes 15 hours to complete. How many hours does it take for Aihui to complete the grading alone?

A. 70 B. 80 C. 90 D. 100 E. 120

6. 若 $x - \frac{1}{x} = 4$，求 $x^3 - \frac{1}{x^3}$ 的值。
If $x - \frac{1}{x} = 4$, find the value of $x^3 - \frac{1}{x^3}$.

A. 52 B. 60 C. 76 D. 80 E. 88

7. 若 a，b 是正的实数，$a+b=200$，$ab=8100$，求 $\lfloor\sqrt{a}+\sqrt{b}\rfloor$。

If a and b are positive real numbers, $a+b=200$, $ab=8100$, find $\lfloor\sqrt{a}+\sqrt{b}\rfloor$.

A. 16 B. 17 C. 18 D. 19 E. 20

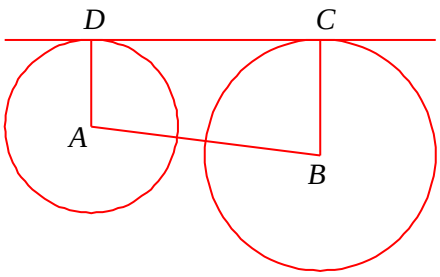
8. 已知 $n=2^{1234567}-1$。求 n 除以 7 的余数。

Given that $n=2^{1234567}-1$. Find the remainder when n is divided by 7.

A. 1 B. 2 C. 3 D. 5 E. 6

9. 下图中，A 及 B 分别为两圆的圆心，直线 CD 分别与两圆相切于 C、D 两点。若两圆的半径分别为 6 及 8，A、B 的距离为 16，求四边形 $ABCD$ 的面积。

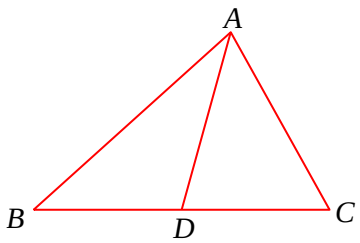
In the figure below, A and B are centers of the two circles. The line CD is tangent to the circles at points C and D. If the radii of the two circles are 6 and 8 respectively, and the distance between A and B is 16, find the area of the quadrilateral $ABCD$.



A. $36\sqrt{7}$ B. $42\sqrt{7}$ C. 111 D. 112 E. $72\sqrt{7}$

10. 下图中，D 是线段 BC 的中点。若 $AB=30$，$AC=16$，$BC=34$，求 AD 的长。

In the figure below, D is the midpoint of BC. If $AB=30$, $AC=16$, $BC=34$, find the length of AD.



A. 14 B. 15 C. 16 D. 17 E. 18

第 11 至第 20 题，问答题，每题 5 分。

Question 11 to Question 20, short questions, each question carries 5 marks.

11. 将 6 粒一样的球分进 3 个一样的箱子中，每个箱子 2 粒，有多少种方法?

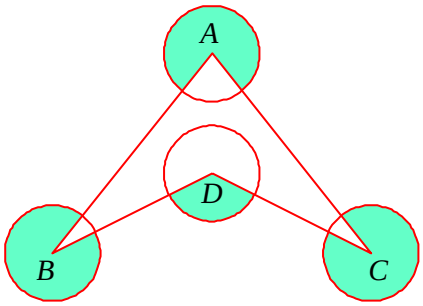
How many ways are there to distribute 6 identical balls into 3 identical boxes so that each box contains 2 balls?

12. 一支原子笔的价钱是 RM 1.30。明华有七张 RM 1.00 的钞票及三张 RM 5.00 的钞票，则他最多可以买到几支原子笔?

The price of one pen is RM 1.30. Ming Hua has seven RM 1.00 notes and three RM 5.00 notes. What is the maximum number of pens that Ming Hua can buy?

13. 下图中，四个等圆的圆心分别在四边形 $ABCD$ 的顶点上。若每一个圆的面积都是 32，求四个阴影区域的面积之和。

In the figure shown below, the centers of the four identical circles are on the vertices of quadrilateral $ABCD$. If the area of each circle is 32, find the sum of the areas of the four shaded regions.



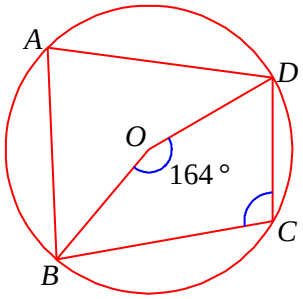
14. 下午 2 点 40 分，时钟上的时针与分针所形成的锐角是 $x°$，求 x。

At 2:40 p.m., the hour-hand and the minute-hand on a clock form an acute angle of $x°$, find x.

15. 若 a，b，c 是正整数使得 $\dfrac{35}{16}=a+\dfrac{1}{b+\dfrac{1}{c}}$，求 $a+2b+3c$ 的值。

If a, b and c are positive integers such that $\dfrac{35}{16}=a+\dfrac{1}{b+\dfrac{1}{c}}$, find the value of $a+2b+3c$.

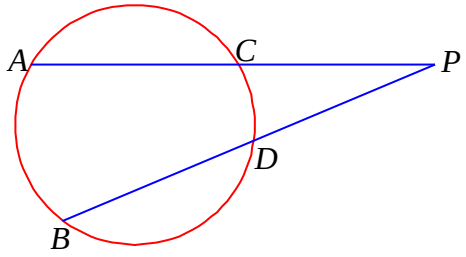
16. 下图中，$ABCD$ 是以 O 为圆心的圆内接四边形，$\angle BOD=164°$。若 $\angle BCD=x°$，求 x。
In the figure below, $ABCD$ is a quadrilateral inscribed in the circle centered at O, $\angle BOD=164°$. If $\angle BCD=x°$, find x.



17. 一个袋子里装有不多于 300 粒的糖果。如果每次拿 3 粒糖果出来，或每次拿 5 粒出来，或每次拿 7 粒出来，结果都剩下 2 粒糖果在袋子中。问袋子里最多可能有几粒糖果?
A bag contains not more than 300 sweets. If the sweets are taken out 3 at a time, or 5 at a time, or 7 at a time, there are always 2 sweets left in the bag. What is the largest possible number of sweets in the bag?

18. 如下图所示，直线 PA 及 PB 分别交圆于 A，C 及 B，D。若 $PC=8$，$AC=6$，$BD=9$，$PD=x$，求 $\lfloor x\rfloor$。

As shown in the figure below, the lines PA and PB intersect the circle at the points A, C and B, D respectively. If $PC=8$, $AC=6$, $BD=9$, $PD=x$, find $\lfloor x\rfloor$.



19. 求函数 $f(x)=\sum_{n=10}^{20}|x-n|=|x-10|+|x-11|+|x-12|+\cdots+|x-20|$ 的最小可能值。

Find the smallest possible value of the function

$$f(x)=\sum_{n=10}^{20}|x-n|=|x-10|+|x-11|+|x-12|+\cdots+|x-20|.$$

20. 已知三个连续正偶数的乘积是 1481088，求这三个偶数的和。

Given that the product of three consecutive positive even numbers is 1481088, find the sum of these three even numbers.

第 21 至第 25 题，问答题，每题 6 分。

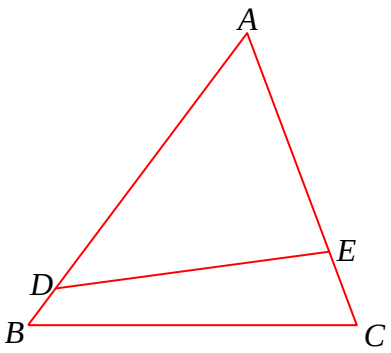
Question 21 to Question 25, short questions, each question carries 6 marks.

21. 已知 x，y 是实数且 $x^2+xy+y^2=10$，求 $2x^2+2xy+y^2-4y+11$ 的最大可能值。

Given that x, y are real numbers such that $x^2+xy+y^2=10$. Find the largest possible value of $2x^2+2xy+y^2-4y+11$.

22. 下图中，D，E 分别是 AB 及 AC 上的点使得 $AD:DB=6:1$，$AE:EC=3:1$。若 $AB=32$，$BC=24$，ΔABC 的面积为 336，求 ΔADE 的面积。

In the figure below, D, E are points on AB and AC respectively such that $AD:DB=6:1$, $AE:EC=3:1$. If $AB=32$, $BC=24$, and the area of ΔABC is 336, find the area of ΔADE.



23. 礼堂里有四个班级的学生在听演讲，他们随意就坐。这四个班级各有 43，46，48 及 51 位学生。主讲者随意邀请一些学生回答问题。主讲者必须邀请至少多少位学生，才能确保其中有 15 位学生来自同一个班级?

Students from four classes are sitting randomly in a hall to listen to a talk. There are respectively 43, 46, 48 and 51 students in these four classes. The speaker randomly invites some students to answer questions. At least how many students should the speaker invite in order to guarantee that 15 of them are from the same class?

24. 已知 a，b 及 c 是两两互质的正整数，且

$$\frac{8a}{b+2c}=\frac{4b}{c+3a}=\frac{3c}{5a+3b}$$

求 b 的值。

Given that a, b and c are pairwise relatively prime positive integers, and

$$\frac{8a}{b+2c}=\frac{4b}{c+3a}=\frac{3c}{5a+3b}$$

Find the value of b.

25. 有多少对整数 (a, b) 满足以下三个条件?

(a) $1 \le a \le 9$;

(b) $0 \le b \le 9$;

(c) 五位数 $\overline{a789b}$ 可以被 12 整除。

How many pairs of integers (a, b) satisfy the following three conditions?

(a) $1 \le a \le 9$;

(b) $0 \le b \le 9$;

(c) The 5-digit number $\overline{a789b}$ is divisible by 12.

第 26 至第 30 题，问答题，每题 8 分。

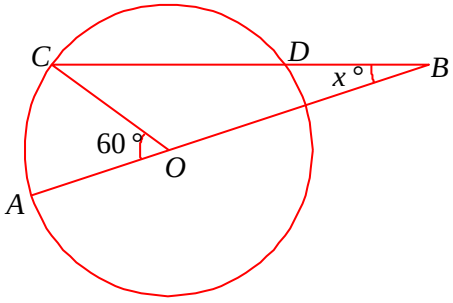
Question 26 to Question 30, short questions, each question carries 8 marks.

26. 有多少个小于 132 的正整数与 132 互质?

How many positive integers less than 132 are relatively prime to 132?

27. 下图中，O 是圆心，A，C，D 是圆上的三点，CDB 与 AOB 是直线，$\angle AOC = 60°$。若 BD 的长等于圆的半径，$\angle ABC = x°$，求 x。

In the figure shown below, A, C, D are three points on the circle centered at O, CDB and AOB are straight lines, $\angle AOC = 60°$. If the length of BD is equal to the radius of the circle, $\angle ABC = x°$, find x.



28. 我们说 320 这个数后面有 1 个 0，174300 这个数后面有两个 0，依此类推。

$$360!=1\times2\times3\times\ldots\times359\times360$$

这个数是由 1 到 360 这 360 个整数的乘积。360! 这个数后面有几个 0?

We say that the number 320 ends with one zero, the number 174300 ends with two zeros, and so on. The number

$$360!=1\times2\times3\times\ldots\times359\times360$$

is the product of 360 integers from 1 to 360. How many zeros does the number 360! end with?

29. 有多少个整数 n 使得 $\dfrac{2n^4+6n^3-3n^2-108n+3}{n+3}$ 也是整数?

How many integers n are there such that $\dfrac{2n^4+6n^3-3n^2-108n+3}{n+3}$ is also an integer?

30. 黄老师要将 6 本不同的书都分出去给三位学生波波，城城及东东，每位学生至少被分得一本，有多少种分法?

Madam Ng would like to give away all 6 different books to three students Bobo, Chengcheng and Dongdong, each student would be getting at least one book. How many ways of distribution are there?