厦门大学马来西亚分校
陈景润杯中学数学比赛

CHEN JINGRUN'S CUP SECONDARY SCHOOL MATHEMATICS COMPETITION 2019

**** 中阶组 ****
INTERMEDIATE CATEGORY

日期：2019 年 4 月 9 日　　时间：上午 10 时至中午 12 时
Date: 9th April 2019　　Time: 10:00 a.m. to 12:00 p.m.

考生须知
Instructions and Information

1. 本试卷共有 30 题。
   This paper contains 30 questions.
   - 第 1 题至第 10 题，选择题， 每题 4 分。
     Question 1 to Question 10, multiple choice questions, each question carries 4 marks.
   - 第 11 题至第 30 题，问答题，每题的答案是一个介于 0 至 1000 之间的整数。
     Question 11 to Question 30, short questions. For each question, the answer is an integer between 0 and 1000.
     - ➢ 第 11 题至第 20 题每题 5 分。
       Question 11 to Question 20, each question carries 5 marks.
     - ➢ 第 21 题至第 25 题每题 6 分。
       Question 21 to Question 25, each question carries 6 marks.
     - ➢ 第 26 题至第 30 题每题 8 分。
       Question 26 to Question 30, each question carries 8 marks.
2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题，必须填写 A，B，C，D 或 E 作为答案。每题只能填入一个答案，否则以答错论。
   Please use 2B pencils to write your answers in the appropriate boxes provided on the answer sheet. For each multiple choice question, please write A, B, C, D or E as answer. If more than one answer is found for a question, no credits would be given for that question.
3. 所有的图形并没有按照比例作图，只作为辅助之用。
   All the diagrams are not drawn to scale. They are intended as aids only.
4. 不许使用计算器，数学工具，手机或其他计算器。
   No calculators, maths stencils, mobile phones or other calculating aids are permitted.
5. 在答案纸上清楚写上姓名，考生编号，学校名称及就读年级。
   Write your name, candidate number, name of school and year of study clearly on the answer sheet.
6. 在监考老师宣布比赛开始之后，才可以翻开此考卷开始作答。
   Do not open this question booklet until you are told to do so.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中，$\lfloor x \rfloor$表示小于或等于 x 的最大整数。

例如：$\lfloor 2 \rfloor = 2$，$\lfloor -2 \rfloor = -2$，$\lfloor 2.6 \rfloor = 2$，$\lfloor -2.6 \rfloor = -3$。

In this paper, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题，选择题，每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 求一正 24 边形每一个内角的度数。

   Find the degree of each internal angle of a regular 24-gon.

   A. $150°$　　B. $156°$　　C. $165°$　　D. $168°$　　E. $170°$

   Each internal angle has degree $\frac{22\times 180°}{24} = 165°$.

   ANSWER:【C】

2. 求 5^{61} 除以 7 的余数。

   Find the remainder when 5^{61} is divided by 7.

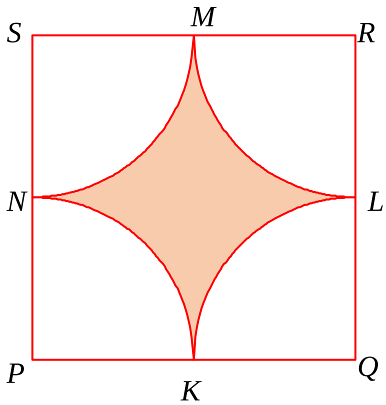
   A. 2　　B. 3　　C. 4　　D. 5　　E. 6

$$5^1 \equiv 5 \bmod 7$$
$$5^2 \equiv 4 \bmod 7$$
$$5^3 \equiv 6 \bmod 7$$
$$5^4 \equiv 2 \bmod 7$$
$$5^5 \equiv 3 \bmod 7$$
$$5^6 \equiv 1 \bmod 7$$
$$5^{61} \equiv 5^{6\times 10+1} \equiv 5 \bmod 7$$

   ANSWER:【D】

3. 下图中，$PQRS$ 是正方形，K，L，M，N 分别是 PQ，QR，RS，SP 的中点，PKN，QKL，RLM，SMN 是扇形。若 $PQ=6$，求阴影部分的面积。

In the figure below, $PQRS$ is a square. K, L, M, N are respectively the midpoints of PQ, QR, RS and SP. PKN, QKL, RLM and SMN are circular sectors. If $PQ=6$, find the area of the shaded region.



A. 27 B. $36-6\pi$ C. $36\pi-36$ D. $18\pi-36$ E. $36-9\pi$

The sum of the areas of the four sectors PKN, QKL, RLM and SMN is the area of a circle with radius 3, which is 9π.

Hence, the area of the shaded region is $36-9\pi$.

ANSWER:【E】

4. 已知 $\sqrt{x+y-4}+\sqrt{2x-y+7}=0$，求 $x+2y$ 的值。

Given that $\sqrt{x+y-4}+\sqrt{2x-y+7}=0$, find the value of $x+2y$.

A. 9 B. 11 C. 13 D. 15 E. 17

Since $\sqrt{x+y-4}\geq 0$ and $\sqrt{2x-y+7}\geq 0$, we find that

$x+y-4=0$ and $2x-y+7=0$

This gives $x=-1$, $y=5$.

Hence, $x+2y=9$.

ANSWER:【A】

5. 已知 ΔABC 是锐角三角形， $AB=15$， $BC=8$， $AC=x$， 且 x 是整数。求 x 的最大可能值。

Given that ΔABC is an acute-angled triangle with $AB=15$, $BC=8$ and $AC=x$. If x is an integer, find the largest possible value of x.

A. 16 B. 17 C. 18 D. 20 E. 22

$x^2=15^2+8^2-2\times15\times8\times\cos B$

Since $0°<\angle B<90°$, we find that $\cos B>0$.

Hence, $x^2<15^2+8^2=17^2$

Therefore, $x<17$.

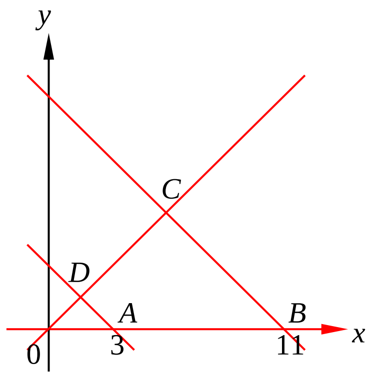
Since x is an integer, the largest possible value of x is 16.

ANSWER:【A】

6. 已知平面上一区域 R 是由四条直线 $x+y=3$， $x+y=11$， $y=0$ 及 $y=x$ 所围成。求此区域的面积。

Given that R is a region in the plane enclosed by four lines $x+y=3$, $x+y=11$, $y=0$ and $y=x$. Find the area of this region.

A. 14 B. $14\sqrt{2}$ C. 28 D. $28\sqrt{2}$ E. 56



The region R is the trapezium $ABCD$.

$AD=\frac{3}{\sqrt{2}}$, $BC=\frac{11}{\sqrt{2}}$, $CD=\frac{8}{\sqrt{2}}$

Hence, the area of the trapezium is $\frac{\frac{3}{\sqrt{2}}+\frac{11}{\sqrt{2}}}{2}\times\frac{8}{\sqrt{2}}=28$.

ANSWER:【C】

7. 介于 201 与 2019 之间的整数，有多少个的个位数是 3?
Among all the integers between 201 and 2019, how many of them end with the digit 3?

A. 181 B. 182 C. 183 D. 184 E. 185

Among the integers between 201 and 2019, those end with digit 3 are
203, 213, 223, $\cdots$, 2013
There are $201-20+1=182$ of them.

ANSWER:【B】

8. 已知 $\frac{x}{x+y+z}=\frac{1}{3}$， $\frac{y}{x+y+z}=\frac{1}{4}$， 求 $\frac{24x+36y+48z}{x+y+z}$。
Given that $\frac{x}{x+y+z}=\frac{1}{3}$ and $\frac{y}{x+y+z}=\frac{1}{4}$. Find $\frac{24x+36y+48z}{x+y+z}$.

A. 34 B. 35 C. 36 D. 37 E. 38

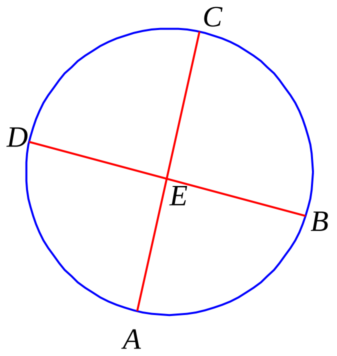
$$\begin{aligned}\frac{z}{x+y+z}&=1-\frac{x}{x+y+z}-\frac{y}{x+y+z}\\&=1-\frac{1}{3}-\frac{1}{4}\\&=\frac{5}{12}\end{aligned}$$

$$\begin{aligned}\frac{24x+36y+48z}{x+y+z}&=24\times\frac{1}{3}+36\times\frac{1}{4}+48\times\frac{5}{12}\\&=8+9+20\\&=37\end{aligned}$$

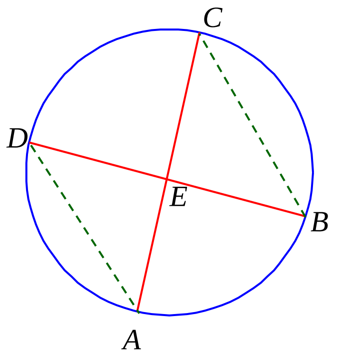
ANSWER:【D】

9. 如下图所示，A，B，C，D是圆上四点，直线AC与BD相交于点E。若$AE=60$，$CE=70$，$BD=134$，$BE>DE$，求BE的长。

As shown in the figure below, A, B, C and D are four points on the circle with lines AC and BD intersecting at the point E. If $AE=60$, $CE=70$, $BD=134$, $BE>DE$, find the length of BE.



A. 68 B. 70 C. 72 D. 80 E. 84



$\angle ADB=\angle ACB$, $\angle DAC=\angle DBC$, $\therefore\ \Delta ADE\sim\Delta BCE$

$$\frac{AE}{BE}=\frac{DE}{CE}$$

If $BE=x$, then $DE=134-x$,

$$x(134-x)=60\times 70=4200$$

$$x^2-134x+4200=0$$

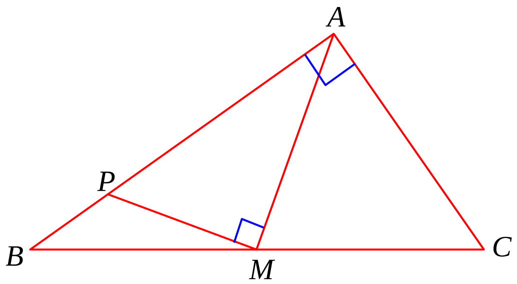
$$(x-50)(x-84)=0$$

Since $BE>DE$, we find that $BE=84$.

ANSWER:【E】

10. 下图中，ΔABC 是直角三角形，$\angle BAC = 90°$。M 是 BC 的中点，P 是 AB 上的一点使得 PM 垂直于 AM。已知 $AB = 96$，$AC = 72$，求 BP 的长。

In the figure below, ΔABC is a right-angled triangle with $\angle BAC = 90°$. M is the midpoint of BC, and P is a point on AB such that PM is perpendicular to AM. Given that $AB = 96$, $AC = 72$, find the length of BP.



A. 20　　B. 21　　C. 22　　D. 23　　E. 24

$AB = 4\times 24$, $AC = 3\times 24$. Hence, $BC = 5\times 24 = 120$.

$AM = \frac{1}{2}BC = 60$.

$\angle MAB = \angle MBA$

Therefore, $\Delta MAP \sim \Delta ABC$.

$$\frac{AP}{BC} = \frac{AM}{AB}$$

$$AP = \frac{60}{96}\times 120$$

$$= 75$$

Hence, $BP = 96 - 75 = 21$.

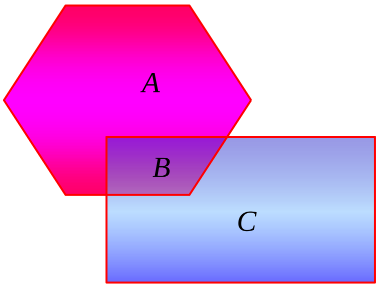
ANSWER:【B】

第 11 至第 20 题，问答题，每题 5 分。

Question 11 to Question 20, short questions, each question carries 5 marks.

11. 下图中，一个六边形 (由区域 A 与区域 B 所组成) 与一个长方形 (由区域 B 与区域 C 所组成) 重叠。重叠的部分 B 的面积占整个六边形面积的 $\frac{3}{20}$，而占整个长方形面积的 $\frac{2}{17}$。若区域 A 的面积与区域 C 的面积之比是 $\frac{m}{n}$，其中 m 与 n 是互质的正整数，求 $m+n$ 的值。

In the figure below, a hexagon (made up of region A and region B) and a rectangle (made up of region B and region C) overlap. The overlapped region B is $\frac{3}{20}$ of the hexagon, and $\frac{2}{17}$ of the rectangle. If the ratio of the region A to region C is $\frac{m}{n}$, where m and n are relatively prime positive integers, find the value of $m+n$.



$S_A : S_B = 17 : 3$

$S_B : S_C = 2 : 15$

Hence, $S_A : S_B : S_C = 34 : 6 : 45$

$$\frac{S_A}{S_C} = \frac{34}{45}$$

$m = 34$, $n = 45$, $m + n = 79$

ANSWER: 【79】

12. 已知 $x=4+\sqrt{13}$， $y=29-4\sqrt{52}$， 求 $\lfloor x^6y^3 \rfloor$ 的值。

Given that $x=4+\sqrt{13}$ and $y=29-4\sqrt{52}$, find the value of $\lfloor x^6y^3 \rfloor$.

Notice that

$$\left(4-\sqrt{13}\right)^2 = 16+13-4\times 2\sqrt{13}$$
$$= 29-4\sqrt{52}$$

Therefore

$$x^6y^3 = \left(4+\sqrt{13}\right)^6\left(4-\sqrt{13}\right)^6$$
$$=(16-13)^6$$
$$=3^6$$
$$=729$$

ANSWER:【729】

13. 求 72 的所有正因数之和。

Find the sum of all positive factors of 72.

$72=2^3\times 3^2$

Expand $\left(1+2+2^2+2^3\right)\left(1+3+3^2\right)$ will give the sum of all positive factors of 72. Hence, the sum is $15\times 13=195$.

ANSWER:【195】

14. 求$1080\times\left(\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}\right)$。

Find $1080\times\left(\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}\right)$.

$$
\begin{aligned}
&1080\times\left(\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}\right)\\
&=540\times\left(\frac{2}{3\times5}+\frac{2}{4\times6}+\frac{2}{5\times7}+\frac{2}{6\times8}+\frac{2}{7\times9}+\frac{2}{8\times10}\right)\\
&=540\times\left(\frac{1}{3}-\frac{1}{5}+\frac{1}{4}-\frac{1}{6}+\frac{1}{5}-\frac{1}{7}+\frac{1}{6}-\frac{1}{8}+\frac{1}{7}-\frac{1}{9}+\frac{1}{8}-\frac{1}{10}\right)\\
&=540\times\left(\frac{1}{3}+\frac{1}{4}-\frac{1}{9}-\frac{1}{10}\right)\\
&=180+135-60-54\\
&=201
\end{aligned}
$$

ANSWER:【201】

15. 有多少种方法可以将 8 支一样的笔分给 3 位学生，每人至少得 2 支?
How many ways are there to distribute 8 identical pens to 3 students so that each student will get at least 2 pens?

First we give each student 2 pens. Then for the remaining 2 pens, we can give both of them to one of the students among the three, which has 3 ways; or we can give the two pens to two of the three students, each one getting one pen, and there are another 3 ways.
Hence, there are 6 ways in total to distribute the pens.

ANSWER:【6】

16. 800 位考生参加考试，有 193 人数学不及格，121 人英语不及格，138 人科学不及格。三科都及格的学生最多有几人?
800 students took part in an examination. 193 of them failed Mathematics, 121 failed English, and 138 failed Science. At most how many students passed all these three subjects?

In the best case scenario, the students that failed English or Science also failed Mathematics. Therefore, at most $800-193=607$ students passed all the three subjects.

ANSWER:【607】

17. 已知 f 是一函数使得对于所有的实数 x， $f(768-x)-f(x)=x-a$，其中 a 是一常数。求 a 的值。

Given that f is a function such that for all real numbers x, $f(768-x)-f(x)=x-a$, where a is a constant. Find the value of a.

Replace x by $768-x$, we find that

$$f(x)-f(768-x)=768-x-a$$

Therefore,

$$x-a=x-768+a$$
$$2a=768$$
$$a=384$$

ANSWER:【384】

18. 有多少对正整数 (x, y) 满足 $5x+13y=1001$?

How many pairs of positive integers (x, y) satisfy $5x+13y=1001$?

$$5x+13y=1001$$
$$13y\equiv 1001 \quad \text{mod } 5$$
$$3y\equiv 1 \quad \text{mod } 5$$

Hence, $y\equiv 2 \quad \text{mod } 5$

$$y=5k+2$$
$$5x=1001-13(5k+2)$$
$$=975-13\times 5k$$
$$x=195-13k$$

Since $x>0$, $y>0$, we find that $k\geq 0$, $k\leq 14$.

There are 15 pairs of positive integers (x, y) that satisfy $5x+13y=1001$.

ANSWER:【15】

19. 若 x 与 y 是正的实数，求

$$f\left(x,y\right)=\left(28x+63y\right)\left(\frac{1}{x}+\frac{1}{y}\right)$$

的最小可能值。

If x and y are positive real numbers, find the smallest possible value of

$$f\left(x,y\right)=\left(28x+63y\right)\left(\frac{1}{x}+\frac{1}{y}\right)$$

$$\begin{aligned}f\left(x,y\right)&=\left(28x+63y\right)\left(\frac{1}{x}+\frac{1}{y}\right)\\&=28+63+28\frac{x}{y}+63\frac{y}{x}\\&\geq 91+2\sqrt{28\frac{x}{y}\times 63\frac{y}{x}}\\&=91+2\times 2\times 7\times 3\\&=175\end{aligned}$$

ANSWER:【175】

20. 已知 x 与 y 是实数且 $x>y$， $x+y=14$， $xy=12$，求 $x^2+\frac{168}{x}$ 的值。

Given that x and y are real numbers such that $x>y$, $x+y=14$ and $xy=12$, find the value of $x^2+\frac{168}{x}$.

$$x\left(14-x\right)=12$$

$$\begin{aligned}x^2&=14x-12\\x^3&=14x^2-12x\\&=14\left(14x-12\right)-12x\\&=184x-168\end{aligned}$$

$$x^2+\frac{168}{x}=184$$

ANSWER:【184】

第 21 至第 25 题，问答题，每题 6 分。

Question 21 to Question 25, short questions, each question carries 6 marks.

21. 已知 a， b， c 都是非零的数，求 $N=128-\frac{a}{|a|}+\frac{b}{|b|}-\frac{c}{|c|}-\frac{ab}{|ab|}+\frac{ac}{|ac|}-\frac{bc}{|bc|}+\frac{abc}{|abc|}$ 的最小可能值。

Given that a, b, c are nonzero numbers, find the smallest possible value of

$$N=128-\frac{a}{|a|}+\frac{b}{|b|}-\frac{c}{|c|}-\frac{ab}{|ab|}+\frac{ac}{|ac|}-\frac{bc}{|bc|}+\frac{abc}{|abc|}.$$

$$\begin{aligned}N&=128-\frac{a}{|a|}+\frac{b}{|b|}-\frac{c}{|c|}-\frac{ab}{|ab|}+\frac{ac}{|ac|}-\frac{bc}{|bc|}+\frac{abc}{|abc|}\\&=127+\left(1-\frac{a}{|a|}\right)\left(1+\frac{b}{|b|}\right)\left(1-\frac{c}{|c|}\right)\end{aligned}$$

For any nonzero x, $\frac{x}{|x|}$ is 1 if x is positive, and $\frac{x}{|x|}$ is -1 if x is negative. Therefore, $1\pm\frac{x}{|x|}$ can only be 0 or 2.

The smallest possible value of N is $127+0=127$, which occurs when a and c are positive, b is negative.

ANSWER:【127】

22. 已知 $a>b>1$ 且 $\frac{1}{\log_a b}+\frac{1}{\log_b a}=\sqrt{10205}$，求 $\frac{1}{\log_{ab} b}-\frac{1}{\log_{ab} a}$ 的值。

Given that $a>b>1$ and $\frac{1}{\log_a b}+\frac{1}{\log_b a}=\sqrt{10205}$, find the value of $\frac{1}{\log_{ab} b}-\frac{1}{\log_{ab} a}$.

$$\begin{aligned} x &= \frac{1}{\log_{ab} b}-\frac{1}{\log_{ab} a} \\ &= \frac{\log ab}{\log b}-\frac{\log ab}{\log a} \\ &= \frac{\log a+\log b}{\log b}-\frac{\log a+\log b}{\log a} \\ &= \frac{\log a}{\log b}-\frac{\log b}{\log a}>0 \end{aligned}$$

$$\frac{\log a}{\log b}+\frac{\log b}{\log a}=\sqrt{10205}$$

$$\begin{aligned} x^2 &= \left(\frac{\log a}{\log b}+\frac{\log b}{\log a}\right)^2-4 \\ &= 10201 \\ &= 101^2 \\ x &= 101 \end{aligned}$$

ANSWER:【101】

23. 求多项式 $f(x)=x^{11}-2019x+2019$ 除以 $x-2$ 的余数。

Find the remainder when the polynomial $f(x)=x^{11}-2019x+2019$ is divided by $x-2$.

Let

$$f(x)=(x-2)q(x)+r$$

where $q(x)$ is the quotient when $f(x)$ is divided by $x-2$, and r is the remainder.

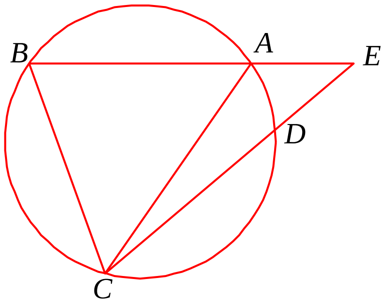
Putting $x=2$, we have

$$\begin{aligned} r &= f(2) \\ &= 2^{11}-2019\times 2+2019 \\ &= 2048-2019 \\ &= 29 \end{aligned}$$

ANSWER:【29】

24. 下图中， A ， B ， C ， D 是圆上的四点， $\overset{\frown}{AB}=\overset{\frown}{BC}=\overset{\frown}{CD}$ 。若 $\angle AEC=44°$ ， $\angle ACD=x°$ ，求 x 。

In the figure below, A, B, C and D are four points on the circle and $\overset{\frown}{AB}=\overset{\frown}{BC}=\overset{\frown}{CD}$. If $\angle AEC=44°$, $\angle ACD=x°$, find x.



$\angle BAC=(44+x)°$.

$\overset{\frown}{AB}=\overset{\frown}{BC} \quad \Rightarrow \quad \angle BCA=\angle BAC=(44+x)°$

$\overset{\frown}{AB}=\overset{\frown}{CD} \quad \Rightarrow \quad AB=CD$

$\angle ABC=\angle DCB=(44+2x)°$

$$44+x+44+x+44+2x=180$$
$$4x=48$$
$$x=12$$

ANSWER:【12】

25. 已知一数列 a_1， a_2， $\cdots$ 的定义为 $a_1=1$，且对于 $n\geq 1$，

$$a_{n+1}=\frac{a_n}{1+2na_n}$$

求 $\frac{1}{a_{2019}}$ 的最后三位数。

Given that a sequence of numbers a_1, a_2, $\cdots$ is defined by $a_1=1$, and for all $n\geq 1$,

$$a_{n+1}=\frac{a_n}{1+2na_n}$$

Find the last three digits of $\frac{1}{a_{2019}}$.

Notice that

$$\frac{1}{a_{n+1}}=\frac{1}{a_n}+2n$$

Hence,

$$\begin{aligned}\frac{1}{a_{2019}}-\frac{1}{a_1}&=\frac{1}{a_{2019}}-\frac{1}{a_{2018}}+\frac{1}{a_{2018}}-\frac{1}{a_{2017}}+\cdots+\frac{1}{a_2}-\frac{1}{a_1}\\&=2\times(2018+2017+\cdots+1)\end{aligned}$$

$$\begin{aligned}\frac{1}{a_{2019}}&=1+2018\times 2019\\&\equiv 1+18\times 19 \mod 1000\\&\equiv 343 \mod 1000\end{aligned}$$

ANSWER:【343】

第 26 至第 30 题，问答题，每题 8 分。

Question 26 to Question 30, short questions, each question carries 8 marks.

26. 若 n 是正整数，求 $\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n$ 的最小可能值。

If n is a positive integer, find the smallest possible value of $\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n$.

If $n \equiv 0 \mod 6$, $n = 6k$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 2k + k + 799 - 6k = 799$.

If $n \equiv 1 \mod 6$, $n = 6k + 1$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 2k + k + 799 - 6k - 1 = 798$.

If $n \equiv 2 \mod 6$, $n = 6k + 2$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 1 + 2k + k + 799 - 6k - 2 = 798$.

If $n \equiv 3 \mod 6$, $n = 6k + 3$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 1 + 2k + 1 + k + 799 - 6k - 3 = 798$.

If $n \equiv 4 \mod 6$, $n = 6k + 4$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 2 + 2k + 1 + k + 799 - 6k - 4 = 798$.

If $n \equiv 5 \mod 6$, $n = 6k + 5$,

$\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{6} \right\rfloor + 799 - n = 3k + 2 + 2k + 1 + k + 799 - 6k - 5 = 797$.

Therefore, the smallest possible value is 797.

ANSWER:【797】

27. 已知a_1，a_2，$\cdots$，a_{10}是正整数且

$$a_1^2+(2a_2)^2+(3a_3)^2+(4a_4)^2+(5a_5)^2+(6a_6)^2+(7a_7)^2+(8a_8)^2+(9a_9)^2+(10a_{10})^2=405$$

求$a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$的最小可能值。

Given that a_1, a_2, $\cdots$, a_{10} are positive integers such that

$$a_1^2+(2a_2)^2+(3a_3)^2+(4a_4)^2+(5a_5)^2+(6a_6)^2+(7a_7)^2+(8a_8)^2+(9a_9)^2+(10a_{10})^2=405$$

Find the smallest possible value of $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$.

$1^2+2^2+3^2+\cdots+10^2=385$

Hence,

$(a_1^2-1)+2^2(a_2^2-1)+\cdots+10^2(a_{10}^2-1)=20$

Since $a_k^2-1\geq 0$ for all $1\leq k\leq 10$, and $a_k^2-1\geq 3$ if $a_k\geq 2$

we find that $a_3=a_4=\cdots=a_{10}=1$, and

$$a_1^2+4a_2^2=25$$

This only has the solution $a_1=3$ and $a_2=2$.

Hence, the smallest possible value of $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$ is 13.

ANSWER:【13】

28. 已知 m 是正整数且方程式 $x^2+2(m+14)x+(120m+99)=0$ 的两个根都是整数，求 m 的最小可能值。

Given that m is a positive integer such that the two roots of the equation $x^2+2(m+14)x+(120m+99)=0$ are both integers, find the smallest possible value of m.

The two roots must be both negative integers. Let them be $-a$ and $-b$, where $a>b>0$. Then

$$a+b=2(m+14)$$
$$ab=120m+99$$

Since $120m+99$ is an odd number, both a and b must be odd numbers. This implies that $c=\frac{a-b}{2}$ must be a positive integer.

$$\begin{aligned}c^2&=\left(\frac{a-b}{2}\right)^2\\&=\left(\frac{a+b}{2}\right)^2-ab\\&=m^2+28m+196-120m-99\\&=m^2-92m+97\\&=(m-46)^2-2019\end{aligned}$$

Hence,

$$(m-46+c)(m-46-c)=2019$$

Since the prime factorization of 2019 is 3×673, and $m-46+c>m-46-c$, we can only have the following cases:

$$\begin{cases}m-46+c=2019\\m-46-c=1\end{cases}$$
$$\begin{cases}m-46+c=-1\\m-46-c=-2019\end{cases}$$
$$\begin{cases}m-46+c=673\\m-46-c=3\end{cases}$$
$$\begin{cases}m-46+c=-3\\m-46-c=-673\end{cases}$$

Since m is a positive integer, m can only be 1056 or 384.

Hence, the smallest possible value of m is 384.

ANSWER:【384】

29. 已知集合 $S=\{1, 2, 3, \cdots, 2019\}$ 是由 1 至 2019 的所有正整数所组成的集合。若任何一个包含 10 个元素的 S 的子集合都包含两个相异的数 a 与 b 使得 $|a-b|\le k$，求 k 的最小可能值。

Given that $S=\{1, 2, 3, \cdots, 2019\}$ is the set that contains all positive integers from 1 to 2019. If any subset of S with 10 elements must contain two distinct numbers a and b with $|a-b|\le k$, find the smallest possible value of k.

Let A be a subset of S that contains 10 elements. Assume that its elements are given by a_1, a_2, $\cdots$, a_{10} in ascending order. If $|a_i-a_j|>m$ for all distinct i and j, then

$$a_{10}-a_9\ge m+1$$
$$a_9-a_8\ge m+1$$
$$\vdots$$
$$a_2-a_1\ge m+1$$

Hence,

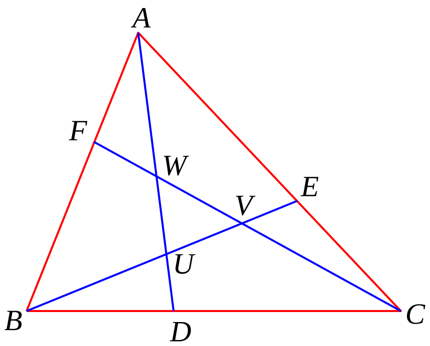
$$2018\ge a_{10}-a_1=9(m+1)$$
$$m+1\le 224$$
$$m\le 223$$

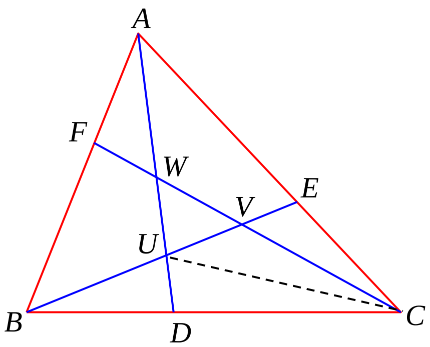
Therefore, the smallest possible value of k is 224.

ANSWER:【224】

30. 如下图所示，ΔABC 中，D、E、F 分别是 BC、CA 及 AB 边上的点，线段 AD 与线段 BE 相交于点 U，线段 BE 与线段 CF 相交于点 V，线段 CF 与线段 AD 相交于点 W，$BU=UE$，$CV=VF$，$AW=WD$。若 $CD=22$，$BD=x+\sqrt{y}$，其中 x 及 y 都是整数，且 y 不是平方数，求 $x+y$ 的值。

As shown in the figure below, in ΔABC, D, E, F are respectively points on BC, CA and AB. The line segment AD intersects the line segment BE at point U, the line segment BE intersects the line segment CF at point V, and the line segment CF intersects the line segment AD at point W. Given that $BU=UE$, $CV=VF$, $AW=WD$ and $CD=22$. If $BD=x+\sqrt{y}$, where x and y are integers and y is not a perfect square, find the value of $x+y$.





Let $\frac{BD}{CD}=\alpha$, $\frac{CE}{EA}=\beta$, $\frac{AF}{FB}=\gamma$.

If the area of ΔCUD, denoted by $S_{\Delta CUD}$ is a, since $\frac{BD}{CD}=\alpha$, then $S_{\Delta BUD}=\alpha a$.

If $S_{\Delta AUB}=b$, since $BU=UE$, then $S_{\Delta AUE}=b$.

Since $BU=UE$, $S_{\Delta CUE}=S_{\Delta BUC}=(\alpha+1)a$.

Now, $\alpha=\frac{BD}{CD}=\frac{S_{\Delta ABD}}{S_{\Delta ACD}}=\frac{b+\alpha a}{b+(\alpha+2)a}$

$$\alpha b+\alpha(\alpha+2)a=b+\alpha a$$
$$\alpha(\alpha+1)a=(1-\alpha)b$$

Hence, $\beta=\frac{CE}{EA}=\frac{S_{\Delta CUE}}{S_{\Delta AUE}}=(\alpha+1)\frac{a}{b}=\frac{1-\alpha}{\alpha}$

Using the same reasoning, we find that $\gamma=\frac{1-\beta}{\beta}$, $\alpha=\frac{1-\gamma}{\gamma}$.

This gives $\gamma=\frac{2\alpha-1}{1-\alpha}$, $\alpha=\frac{2-3\alpha}{2\alpha-1}$.

From this, it follows that $2\alpha^2+2\alpha-2=0$

$$\alpha=\frac{-1+\sqrt{5}}{2}$$

$$x+\sqrt{y}=22\times\frac{-1+\sqrt{5}}{2}=-11+\sqrt{605}$$

Hence, $x=-11$, $y=605$, $x+y=594$.

ANSWER:【594】