厦门大学马来西亚分校

陈景润杯中学数学比赛

CHEN JINGRUN'S CUP SECONDARY SCHOOL MATHEMATICS COMPETITION 2019

**** 中阶组 ****

INTERMEDIATE CATEGORY

日期：2019 年 4 月 9 日　　　　时间：上午 10 时至中午 12 时

Date: 9th April 2019　　　　Time: 10:00 a.m. to 12:00 p.m.

考生须知

Instructions and Information

1. 本试卷共有 30 题。
   This paper contains 30 questions.
   - 第 1 题至第 10 题，选择题， 每题 4 分。
     Question 1 to Question 10, multiple choice questions, each question carries 4 marks.
   - 第 11 题至第 30 题，问答题，每题的答案是一个介于 0 至 1000 之间的整数。
     Question 11 to Question 30, short questions. For each question, the answer is an integer between 0 and 1000.
     - ➢ 第 11 题至第 20 题每题 5 分。
       Question 11 to Question 20, each question carries 5 marks.
     - ➢ 第 21 题至第 25 题每题 6 分。
       Question 21 to Question 25, each question carries 6 marks.
     - ➢ 第 26 题至第 30 题每题 8 分。
       Question 26 to Question 30, each question carries 8 marks.
2. 请在答案纸内适当的空格中用 2B 铅笔清楚的写出每题的答案。对于选择题，必须填写 A，B，C，D 或 E 作为答案。每题只能填入一个答案，否则以答错论。
   Please use 2B pencils to write your answers in the appropriate boxes provided on the answer sheet. For each multiple choice question, please write A, B, C, D or E as answer. If more than one answer is found for a question, no credits would be given for that question.
3. 所有的图形并没有按照比例作图，只作为辅助之用。
   All the diagrams are not drawn to scale. They are intended as aids only.
4. 不许使用计算器，数学工具，手机或其他计算器。
   No calculators, maths stencils, mobile phones or other calculating aids are permitted.
5. 在答案纸上清楚写上姓名，考生编号，学校名称及就读年级。
   Write your name, candidate number, name of school and year of study clearly on the answer sheet.
6. 在监考老师宣布比赛开始之后，才可以翻开此考卷开始作答。
   Do not open this question booklet until you are told to do so.

~~ 说明 ~~

~~ Notes ~~

在这份试卷中，$\lfloor x \rfloor$表示小于或等于x的最大整数。

例如：$\lfloor 2 \rfloor = 2$，$\lfloor -2 \rfloor = -2$，$\lfloor 2.6 \rfloor = 2$，$\lfloor -2.6 \rfloor = -3$。

In this paper, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

For example, $\lfloor 2 \rfloor = 2$, $\lfloor -2 \rfloor = -2$, $\lfloor 2.6 \rfloor = 2$, $\lfloor -2.6 \rfloor = -3$.

第 1 至第 10 题，选择题，每题 4 分。

Question 1 to Question 10, multiple choice questions, each question carries 4 marks.

1. 求一正 24 边形每一个内角的度数。

   Find the degree of each internal angle of a regular 24-gon.

   A. 150° B. 156° C. 165° D. 168° E. 170°

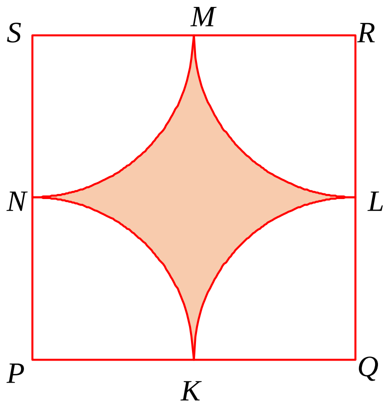
2. 求5^{61}除以 7 的余数。

   Find the remainder when 5^{61} is divided by 7.

   A. 2 B. 3 C. 4 D. 5 E. 6

3. 下图中，$PQRS$是正方形，K，L，M，N分别是PQ，QR，RS，SP的中点，PKN，QKL，RLM，SMN是扇形。若$PQ=6$，求阴影部分的面积。

In the figure below, $PQRS$ is a square. K, L, M, N are respectively the midpoints of PQ, QR, RS and SP. PKN, QKL, RLM and SMN are circular sectors. If $PQ=6$, find the area of the shaded region.



A. 27 B. $36-6\pi$ C. $36\pi-36$ D. $18\pi-36$ E. $36-9\pi$

4. 已知$\sqrt{x+y-4}+\sqrt{2x-y+7}=0$，求$x+2y$的值。

Given that $\sqrt{x+y-4}+\sqrt{2x-y+7}=0$, find the value of $x+2y$.

A. 9 B. 11 C. 13 D. 15 E. 17

5. 已知ΔABC是锐角三角形，$AB=15$，$BC=8$，$AC=x$，且x是整数。求x的最大可能值。

Given that ΔABC is an acute-angled triangle with $AB=15$, $BC=8$ and $AC=x$. If x is an integer, find the largest possible value of x.

A. 16 B. 17 C. 18 D. 20 E. 22

6. 已知平面上一区域 R 是由四条直线 $x+y=3$， $x+y=11$， $y=0$ 及 $y=x$ 所围成。求此区域的面积。
Given that R is a region in the plane enclosed by four lines $x+y=3$, $x+y=11$, $y=0$ and $y=x$. Find the area of this region.

A. 14 B. $14\sqrt{2}$ C. 28 D. $28\sqrt{2}$ E. 56

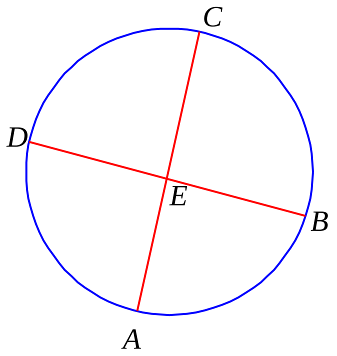
7. 介于 201 与 2019 之间的整数，有多少个的个位数是 3?
Among all the integers between 201 and 2019, how many of them end with the digit 3?

A. 181 B. 182 C. 183 D. 184 E. 185

8. 已知 $\dfrac{x}{x+y+z}=\dfrac{1}{3}$， $\dfrac{y}{x+y+z}=\dfrac{1}{4}$， 求 $\dfrac{24x+36y+48z}{x+y+z}$。
Given that $\dfrac{x}{x+y+z}=\dfrac{1}{3}$ and $\dfrac{y}{x+y+z}=\dfrac{1}{4}$. Find $\dfrac{24x+36y+48z}{x+y+z}$.

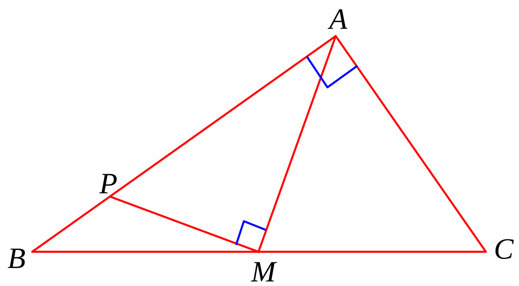
A. 34 B. 35 C. 36 D. 37 E. 38

9. 如下图所示，A，B，C，D是圆上四点，直线AC与BD相交于点E。若$AE=60$，$CE=70$，$BD=134$，$BE>DE$，求BE的长。
As shown in the figure below, A, B, C and D are four points on the circle with lines AC and BD intersecting at the point E. If $AE=60$, $CE=70$, $BD=134$, $BE>DE$, find the length of BE.



A. 68　　B. 70　　C. 72　　D. 80　　E. 84

10. 下图中，ΔABC是直角三角形，$\angle BAC=90°$。M是BC的中点，P是AB上的一点使得PM垂直于AM。已知$AB=96$，$AC=72$，求BP的长。
In the figure below, ΔABC is a right-angled triangle with $\angle BAC=90°$. M is the midpoint of BC, and P is a point on AB such that PM is perpendicular to AM. Given that $AB=96$, $AC=72$, find the length of BP.

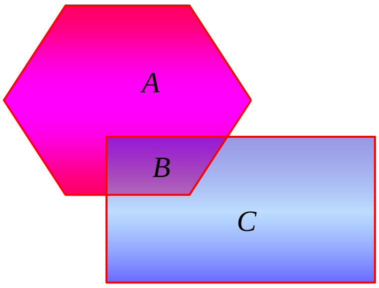


A. 20　　B. 21　　C. 22　　D. 23　　E. 24

第 11 至第 20 题，问答题，每题 5 分。
Question 11 to Question 20, short questions, each question carries 5 marks.

11. 下图中，一个六边形 (由区域 A 与区域 B 所组成) 与一个长方形 (由区域 B 与区域 C 所组成) 重叠。重叠的部分 B 的面积占整个六边形面积的 $\frac{3}{20}$，而占整个长方形面积的 $\frac{2}{17}$。若区域 A 的面积与区域 C 的面积之比是 $\frac{m}{n}$，其中 m 与 n 是互质的正整数，求 $m+n$ 的值。

    In the figure below, a hexagon (made up of region A and region B) and a rectangle (made up of region B and region C) overlap. The overlapped region B is $\frac{3}{20}$ of the hexagon, and $\frac{2}{17}$ of the rectangle. If the ratio of the region A to region C is $\frac{m}{n}$, where m and n are relatively prime positive integers, find the value of $m+n$.



12. 已知 $x=4+\sqrt{13}$，$y=29-4\sqrt{52}$，求 $\lfloor x^6y^3 \rfloor$ 的值。

    Given that $x=4+\sqrt{13}$ and $y=29-4\sqrt{52}$, find the value of $\lfloor x^6y^3 \rfloor$.

13. 求 72 的所有正因数之和。

    Find the sum of all positive factors of 72.

14. 求 $1080\times\left(\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}\right)$。

    Find $1080\times\left(\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\frac{1}{48}+\frac{1}{63}+\frac{1}{80}\right)$.

15. 有多少种方法可以将 8 支一样的笔分给 3 位学生，每人至少得 2 支?
How many ways are there to distribute 8 identical pens to 3 students so that each student will get at least 2 pens?

16. 800 位考生参加考试，有 193 人数学不及格，121 人英语不及格，138 人科学不及格。三科都及格的学生最多有几人?
800 students took part in an examination. 193 of them failed Mathematics, 121 failed English, and 138 failed Science. At most how many students passed all these three subjects?

17. 已知 f 是一函数使得对于所有的实数 x， $f(768-x)-f(x)=x-a$，其中 a 是一常数。求 a 的值。
Given that f is a function such that for all real numbers x, $f(768-x)-f(x)=x-a$, where a is a constant. Find the value of a.

18. 有多少对正整数 (x, y) 满足 $5x+13y=1001$?
How many pairs of positive integers (x, y) satisfy $5x+13y=1001$?

19. 若 x 与 y 是正的实数，求
$$f(x,y)=(28x+63y)\left(\frac{1}{x}+\frac{1}{y}\right)$$
的最小可能值。
If x and y are positive real numbers, find the smallest possible value of
$$f(x,y)=(28x+63y)\left(\frac{1}{x}+\frac{1}{y}\right)$$

20. 已知 x 与 y 是实数且 $x>y$， $x+y=14$， $xy=12$，求 $x^2+\dfrac{168}{x}$ 的值。
Given that x and y are real numbers such that $x>y$, $x+y=14$ and $xy=12$, find the value of $x^2+\dfrac{168}{x}$.

第 21 至第 25 题，问答题，每题 6 分。

Question 21 to Question 25, short questions, each question carries 6 marks.

21. 已知 a，b，c 都是非零的数，求 $N=128-\frac{a}{|a|}+\frac{b}{|b|}-\frac{c}{|c|}-\frac{ab}{|ab|}+\frac{ac}{|ac|}-\frac{bc}{|bc|}+\frac{abc}{|abc|}$ 的最小可能值。

Given that a, b, c are nonzero numbers, find the smallest possible value of

$$N=128-\frac{a}{|a|}+\frac{b}{|b|}-\frac{c}{|c|}-\frac{ab}{|ab|}+\frac{ac}{|ac|}-\frac{bc}{|bc|}+\frac{abc}{|abc|}.$$

22. 已知 $a>b>1$ 且 $\frac{1}{\log_a b}+\frac{1}{\log_b a}=\sqrt{10205}$，求 $\frac{1}{\log_{ab} b}-\frac{1}{\log_{ab} a}$ 的值。

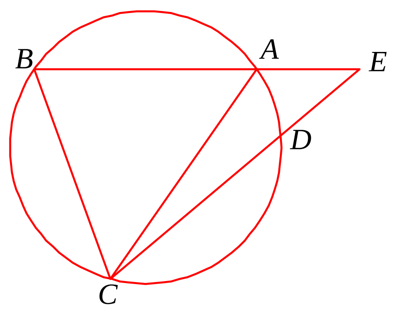
Given that $a>b>1$ and $\frac{1}{\log_a b}+\frac{1}{\log_b a}=\sqrt{10205}$, find the value of $\frac{1}{\log_{ab} b}-\frac{1}{\log_{ab} a}$.

23. 求多项式 $f(x)=x^{11}-2019x+2019$ 除以 $x-2$ 的余数。

Find the remainder when the polynomial $f(x)=x^{11}-2019x+2019$ is divided by $x-2$.

24. 下图中，A，B，C，D 是圆上的四点，$\overset{\frown}{AB}=\overset{\frown}{BC}=\overset{\frown}{CD}$。若 $\angle AEC=44^\circ$，$\angle ACD=x^\circ$，求 x。

In the figure below, A, B, C and D are four points on the circle and $\overset{\frown}{AB}=\overset{\frown}{BC}=\overset{\frown}{CD}$. If $\angle AEC=44^\circ$, $\angle ACD=x^\circ$, find x.



25. 已知一数列a_1，a_2，$\cdots$的定义为$a_1=1$，且对于$n\geq 1$，

$$a_{n+1}=\frac{a_n}{1+2na_n}$$

求$\frac{1}{a_{2019}}$的最后三位数。

Given that a sequence of numbers a_1, a_2, $\cdots$ is defined by $a_1=1$, and for all $n\geq 1$,

$$a_{n+1}=\frac{a_n}{1+2na_n}$$

Find the last three digits of $\frac{1}{a_{2019}}$.

第26至第30题，问答题，每题8分。

Question 26 to Question 30, short questions, each question carries 8 marks.

26. 若n是正整数，求$\left\lfloor \frac{n}{2} \right\rfloor+\left\lfloor \frac{n}{3} \right\rfloor+\left\lfloor \frac{n}{6} \right\rfloor+799-n$的最小可能值。

If n is a positive integer, find the smallest possible value of $\left\lfloor \frac{n}{2} \right\rfloor+\left\lfloor \frac{n}{3} \right\rfloor+\left\lfloor \frac{n}{6} \right\rfloor+799-n$.

27. 已知a_1，a_2，$\cdots$，a_{10}是正整数且

$$a_1^2+(2a_2)^2+(3a_3)^2+(4a_4)^2+(5a_5)^2+(6a_6)^2+(7a_7)^2+(8a_8)^2+(9a_9)^2+(10a_{10})^2=405$$

求$a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$的最小可能值。

Given that a_1, a_2, $\cdots$, a_{10} are positive integers such that

$$a_1^2+(2a_2)^2+(3a_3)^2+(4a_4)^2+(5a_5)^2+(6a_6)^2+(7a_7)^2+(8a_8)^2+(9a_9)^2+(10a_{10})^2=405$$

Find the smallest possible value of $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}$.

28. 已知 m 是正整数且方程式 $x^2+2(m+14)x+(120m+99)=0$ 的两个根都是整数，求 m 的最小可能值。

Given that m is a positive integer such that the two roots of the equation $x^2+2(m+14)x+(120m+99)=0$ are both integers, find the smallest possible value of m.

29. 已知集合 $S=\{1,2,3,\cdots,2019\}$ 是由 1 至 2019 的所有正整数所组成的集合。若任何一个包含 10 个元素的 S 的子集合都包含两个相异的数 a 与 b 使得 $|a-b|\le k$，求 k 的最小可能值。

Given that $S=\{1,2,3,\cdots,2019\}$ is the set that contains all positive integers from 1 to 2019. If any subset of S with 10 elements must contain two distinct numbers a and b with $|a-b|\le k$, find the smallest possible value of k.

30. 如下图所示，ΔABC 中，D、E、F 分别是 BC、CA 及 AB 边上的点，线段 AD 与线段 BE 相交于点 U，线段 BE 与线段 CF 相交于点 V，线段 CF 与线段 AD 相交于点 W，$BU=UE$，$CV=VF$，$AW=WD$。若 $CD=22$，$BD=x+\sqrt{y}$，其中 x 及 y 都是整数，且 y 不是平方数，求 $x+y$ 的值。

As shown in the figure below, in ΔABC, D, E, F are respectively points on BC, CA and AB. The line segment AD intersects the line segment BE at point U, the line segment BE intersects the line segment CF at point V, and the line segment CF intersects the line segment AD at point W. Given that $BU=UE$, $CV=VF$, $AW=WD$ and $CD=22$. If $BD=x+\sqrt{y}$, where x and y are integers and y is not a perfect square, find the value of $x+y$.